Notes on the function gsw_specvol_anom(SA,CT,p)

Specific volume anomaly is defined by Eqn. (3.7.3) of IOC *et al*. (2010), namely

$$\delta\left(S_{\mathrm{A}},\Theta,p\right) = \hat{v}\left(S_{\mathrm{A}},\Theta,p\right) - \hat{v}\left(\tilde{\tilde{S}}_{\mathrm{A}},\tilde{\tilde{\Theta}},p\right), \qquad (1)$$

where the reference values of Absolute Salinity and Conservative Temperature, $\tilde{\tilde{S}}_{\mathrm{A}}$ and $\tilde{\tilde{\Theta}}$, are $S_{\mathrm{SO}} = 35.165\,04\ \mathrm{g\ kg^{-1}}$ and $0°\mathrm{C}$ respectively.

This function, **gsw_specvol_anom**(SA,CT,p) evaluates the specific volume anomaly of seawater as a function of Absolute Salinity, Conservative Temperature and pressure using the 48-term expression, $\hat{v}\left(S_{\mathrm{A}},\Theta,p\right)$. This 48-term rational function expression for density and specific volume is discussed in McDougall and Barker (2013) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al*. (2010)).

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

McDougall T. J., and P. M. Barker, 2013: A computationally efficient 48-term expression for the density of seawater in terms of Conservative Temperature, and related properties of seawater. To be submitted to *Journal of Atmospheric ans Oceanic Technology*.

Here follows section 3.7 of the TEOS-10 manual (IOC *et al*. (2010)).

3.7 Specific volume anomaly

The specific volume anomaly δ is defined as the difference between the specific volume and a given function of pressure. Traditionally δ has been defined as

$$\delta\left(S_{\mathrm{A}},t,p\right) = v\left(S_{\mathrm{A}},t,p\right) - v\left(S_{\mathrm{SO}},0°\mathrm{C},p\right) \qquad (3.7.1)$$

(where the traditional value of Practical Salinity of 35 has been updated to an Absolute Salinity of $S_{\mathrm{SO}} = 35u_{PS} = 35.16504\ \mathrm{g\,kg^{-1}}$ in the present formulation). Note that the second term, $v\left(S_{\mathrm{SO}},0°\mathrm{C},p\right)$, is a function only of pressure. In order to have a surface of constant specific volume anomaly more accurately approximate neutral tangent planes (see section 3.11), it is advisable to replace the arguments S_{SO} and $0°\mathrm{C}$ with more general values $\hat{\hat{S}}_{\mathrm{A}}$ and $\hat{\hat{t}}$ that are carefully chosen (as say the median values of Absolute Salinity and temperature along the surface) so that the more general definition of specific volume anomaly is

$$\hat{\hat{\delta}}\left(S_{\mathrm{A}},t,p\right) = v\left(S_{\mathrm{A}},t,p\right) - v\left(\hat{\hat{S}}_{\mathrm{A}},\hat{\hat{t}},p\right) = g_{P}\left(S_{\mathrm{A}},t,p\right) - g_{P}\left(\hat{\hat{S}}_{\mathrm{A}},\hat{\hat{t}},p\right). \qquad (3.7.2)$$

The last terms in Eqns. (3.7.1) and (3.7.2) are simply functions of pressure and one has the freedom to choose any other function of pressure in its place and still retain the dynamical properties of specific volume anomaly. In particular, one can construct specific volume and enthalpy to be functions of Conservative Temperature (rather than *in situ* temperature) as $\hat{v}(S_A, \Theta, p)$ and $\hat{h}(S_A, \Theta, p)$ and write a slightly different definition of specific volume anomaly as

$$\tilde{\tilde{\delta}}(S_A, \Theta, p) = \hat{v}(S_A, \Theta, p) - \hat{v}\left(\tilde{\tilde{S}}_A, \tilde{\tilde{\Theta}}, p\right) = \hat{h}_P(S_A, \Theta, p) - \hat{h}_P\left(\tilde{\tilde{S}}_A, \tilde{\tilde{\Theta}}, p\right). \tag{3.7.3}$$

This is the form of specific volume anomaly adopted in the GSW Oceanographic Toolbox where the default values of the reference values $\tilde{\tilde{S}}_A$ and $\tilde{\tilde{\Theta}}$ are $S_{SO} = 35.165\,04 \text{ g kg}^{-1}$ and $0°\text{C}$ respectively. The same can also be done with potential temperature so that in terms of the specific volume $\tilde{v}(S_A, \theta, p)$ and enthalpy $\tilde{h}(S_A, \theta, p)$ we can write another form of the specific volume anomaly as

$$\tilde{\tilde{\delta}}(S_A, \theta, p) = \tilde{v}(S_A, \theta, p) - \tilde{v}\left(\tilde{\tilde{S}}_A, \tilde{\tilde{\theta}}, p\right) = \tilde{h}_P(S_A, \theta, p) - \tilde{h}_P\left(\tilde{\tilde{S}}_A, \tilde{\tilde{\theta}}, p\right). \tag{3.7.4}$$

These expressions exploit the fact that (see appendix A.11)

$$\partial h/\partial P\big|_{S_A, \eta} = \partial h/\partial P\big|_{S_A, \Theta} = \partial h/\partial P\big|_{S_A, \theta} = v\,. \tag{3.7.5}$$