7. The 48-term expression $\hat{\rho}(S_A, \Theta, p)$ for density

The computationally efficient 48-term rational function expression for density, $\hat{\rho}(S_A, \Theta, p)$, is described in McDougall *et al.* (2013), in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al.* (2010)), and is the function **gsw_rho**(SA,CT,p) in the GSW Oceanographic Toolbox. Seawater density data was fitted in a "funnel" of data points in (S_A, Θ, p) space which is described in more detail in McDougall *et al.* (2013). The "funnel" extends to a pressure of 8000 dbar. At the sea surface the "funnel" covers the full range of temperature and salinity while for pressures greater than 6500 dbar, the maximum Conservative Temperature of the fitted data is 10°C and the minimum Absolute Salinity is 30 g kg^{-1}. That is, the 48-term fit has been performed over a region of parameter space which includes water that is approximately 10°C warmer and 5 g kg^{-1} fresher in the deep ocean than the seawater which exists in the present ocean.

The rms error of this 48-term approximation to the TEOS-10 density over the oceanographic "funnel" is $0.00046 \text{ kg m}^{-3}$; this can be compared with the rms uncertainty of 0.004 kg m^{-3} of the underlying laboratory density data to which the TEOS-10 Gibbs function was fitted. Similarly, the appropriate thermal expansion coefficient,

$$\alpha^{\Theta} = -\frac{1}{\rho} \frac{\partial \rho}{\partial \Theta}\bigg|_{S_A, p},$$

of the 48-term equation of state is different from the same thermal expansion coefficient evaluated directly from TEOS-10 with an rms error in the "funnel" of $0.069 x 10^{-6} \text{ K}^{-1}$, compared with the rms error of the thermal expansion coefficient of the laboratory data to which the Feistel (2008) Gibbs function was fitted of $0.73 x 10^{-6} \text{ K}^{-1}$. In terms of the evaluation of density gradients, the haline contraction coefficient evaluated from the 48-term equation is many times more accurate than the thermal expansion coefficient.

In dynamical oceanography it is the thermal expansion and haline contraction coefficients α^{Θ} and β^{Θ} which are the most important aspects of the equation of state since the "thermal wind" is proportional to $\alpha^{\Theta}\nabla_p \Theta - \beta^{\Theta}\nabla_p S_A$ and the vertical static stability is given in terms of the buoyancy frequency N by $g^{-1}N^2 = \alpha^{\Theta}\Theta_z - \beta^{\Theta}(S_A)_z$. Hence for dynamical oceanography the 48-term rational function expression for density retains essentially the full accuracy of TEOS-10. The use of the 48-term expression for density has several advantages over using the exact formulation, namely

- it is a function of Conservative Temperature, so eliminating the need to be continually converting between Conservative and *in-situ* temperatures in order to evaluate density,
- it is computationally faster (by a factor of 6.5) to use the 48-term expression, **gsw_rho**(SA,CT,p) (or equivalently **gsw_rho_CT**(SA,CT,p)), rather than using **gsw_rho_CT_exact**(SA,CT,p) which is based on the sum of the Gibbs functions of pure water (IAPWS-09) and of sea salt (IAPWS-08),
- ocean models will use this 48-term equation of state, and it is advantageous for the fields of observational and theoretical oceanography to use the same equation of state as ocean models.

The functions of the GSW Oceanographic Toolbox are listed on the next two pages, after which section 8 begins on page 16.