Notes on the function gsw_pot_rho_t_exact(SA,t,p,pr)

This function, \( \text{gsw\_pot\_rho\_t\_exact}(SA,t,p,pr) \), evaluates the potential density with respect to reference pressure \( p_r \) for given input values of Absolute Salinity \( S_A \), in situ temperature \( t \), and pressure \( p \). This function uses the full TEOS-10 Gibbs function \( g(S_A,t,p) \) of IOC et al. (2010), being the sum of the IAPWS-09 and IAPWS-08 Gibbs functions. The potential density is evaluated directly from the Gibbs function using Eqn. (3.4.2) of the TEOS-10 Manual (IOC et al., 2010), repeated here,

\[
\rho^0(S_A,t,p,p_r) = \rho(S_A,\theta[S_A,t,p,p_r]) = g_p^{-1}(S_A,\theta[S_A,t,p,p_r],p_r). \tag{3.4.2}
\]

This function \( \text{gsw\_pot\_rho\_t\_exact}(SA,t,p,pr) \), amounts to the following two calls to other GSW functions

\[
\begin{align*}
pt &= \text{gsw\_pt\_from\_t}(SA,t,p,pr); \\
pot\_rho\_t\_exact &= \text{gsw\_rho\_t\_exact}(SA,pt,pr);
\end{align*}
\]

References


Here follows section 3.4 of the TEOS-10 Manual (IOC et al., 2010).

3.4 Potential density

Potential density \( \rho^0 \) is the density that a fluid parcel would have if its pressure were changed to a fixed reference pressure \( p_r \) in an isentropic and isohaline manner. Potential density referred to reference pressure \( p_r \) can be written as the pressure integral of the isentropic compressibility \( \kappa \) as

\[
\rho^0(S_A,t,p,p_r) = \rho(S_A,t,p) + \int_p^{p_r} \rho(S_A,\theta[S_A,t,p,p'],p') \kappa(S_A,\theta[S_A,t,p,p'],p') dp'. \tag{3.4.1}
\]

The simpler expression for potential density in terms of the Gibbs function is

\[
\rho^0(S_A,t,p,p_r) = \rho(S_A,\theta[S_A,t,p,p_r]) = g_p^{-1}(S_A,\theta[S_A,t,p,p_r],p_r). \tag{3.4.2}
\]

Using the functional forms of Eqn. (2.8.2) and (2.8.3) for in situ density, that is, either \( \rho = \hat{\rho}(S_A,\theta,p) \) or \( \rho = \hat{\rho}(S_A,\Theta,p) \), potential density with respect to reference pressure \( p_r \) (e.g., 1000 dbar) can be easily evaluated as
\[
\rho^\theta(S_A,t,p,p_t) = \rho^\theta(S_A,t,p,p_t) = \hat{\rho}(S_A,\eta, p_t) = \hat{\rho}(S_A,\Theta, p_t) = \hat{\rho}(S_A,\Theta, p_t),
\]

(3.4.3)

where we note that the potential temperature \( \theta \) in the penultimate expression is the potential temperature with respect to 0 dbar. Once the reference pressure is fixed, potential density is a function only of Absolute Salinity and Conservative Temperature (or equivalently, of Absolute Salinity and potential temperature). Note that it is equally correct to label potential density as \( \rho^\theta \) or \( \rho^\Theta \) (or indeed as \( \rho^\eta \)) because \( \eta \), \( \theta \) and \( \Theta \) are constant during the isentropic and isohaline pressure change from \( p \) to \( p_t \); that is, these variables possess the “potential” property of appendix A.9.

Following the discussion after Eqn. (2.8.2) above, potential density may also be expressed in terms of the pressure derivative of the expressions \( h = \hat{h}(S_A,\theta, p) \) and \( h = \hat{h}(S_A,\Theta, p) \) for enthalpy as (see also appendix P)

\[
\rho^\theta(S_A,t,p,p_t) = \rho^\theta(S_A,t,p,p_t) = \left[ \hat{h}_p(S_A,\theta, p=p_t) \right]^{-1} = \left[ \hat{h}_p(S_A,\Theta, p=p_t) \right]^{-1}.
\]

(3.4.4)