Notes on the function gsw_osmotic_coefficient_t_exact(SA,t,p)

This function, \texttt{gsw_osmotic_coefficient_t_exact}(SA,t,p) evaluates the osmotic coefficient \( \phi \) of seawater from Eqn. (2.14.1) of the TEOS-10 Manual (IOC \textit{et al.}, 2010), namely

\[
\phi = \phi \left( S_A, t, p \right) = - \left[ g \left( S_A, t, p \right) - g \left( 0, t, p \right) - S_A \frac{\partial g}{\partial S_A} \right] \left( m_{SW} R T_0 + t \right)^{-1}. \quad (2.14.1)
\]

where \( m_{SW} \) is the molality of seawater which can be evaluated from the GSW function \texttt{gsw_molality_from_SA}(SA) from the definition given by Eqn. (3.40.10) of the TEOS-10 Manual (IOC \textit{et al.} (2010))

\[
m_{SW} = \frac{S_A}{(1 - S_A) M_S}. \quad (3.40.10)
\]

Here \( M_S \) is the mole-weighted average atomic weight of the elements of sea salt. The paper which defines the Reference-Composition Salinity Scale, Millero \textit{et al.} (2008), derives \( M_S \) to be the value

\[
M_S = 31.403 \, 821 \, 8... \, \text{g mol}^{-1} = 0.031 \, 403 \, 821 \, 8... \, \text{kg mol}^{-1}, \quad (1)
\]

and this value is a fundamental constant of TEOS-10 and can be found by calling \texttt{gsw_atomic_weight}. In Eqn. (3.40.10) Absolute Salinity \( S_A \) must be in units of kg kg\(^{-1}\) which means that \( M_S \) must be in units of kg mol\(^{-1}\) in this equation. Molality \( m_{SW} \) is given by the GSW function \texttt{gsw_molality_from_SA}(SA) in units of mol kg\(^{-1}\).

This function \texttt{gsw_osmotic_coefficient_t_exact}(SA,t,p) uses the full TEOS-10 Gibbs function \( g \left( S_A, t, p \right) \) of IOC \textit{et al.} (2010), being the sum of the IAPWS-09 and IAPWS-08 Gibbs functions. The molality \( m_{SW} \) tends to zero as Absolute Salinity tends to zero, and this would seem to present a problem for the evaluation of the osmotic coefficient from Eqn. (2.14.1). However the combination of the salty part of the Gibbs function and its salinity derivative in this equation also tends to zero linearly as Absolute Salinity tends to zero. The potential singularity at \( S_A = 0 \) kg kg\(^{-1}\) is avoided in this function by carefully collecting all the powers of \( S_A \) in Eqn. (2.14.1) and mathematically cancelling the leading power of Absolute Salinity. Because of this, \texttt{gsw_osmotic_coefficient_t_exact}(SA,t,p) is well-behaved at \( S_A = 0 \) kg kg\(^{-1}\) where in fact the value 1.0 is returned.

Strictly speaking, Eqn. (2.14.1) applies only to seawater of Reference Composition because the value of \( M_S \) used in this equation is the value for Reference-Composition seawater. The value of the mole-weighted average atomic weight of the elements of seawater of arbitrary composition is unknown. If the osmotic coefficient of seawater of general composition is required to an accuracy of better than 0.1\% it is suggested that you contact the authors for further guidance.
2.14 Osmotic coefficient

The osmotic coefficient of seawater $\phi$ is given by

$$\phi = \phi(S_A,t,p) = - \left( g(S_A,t,p) - g(0,t,p) - S_A \frac{\partial g}{\partial S_A} \right) m_{SW} R (T_0 + t)^{-1}. \quad (2.14.1)$$

The osmotic coefficient of seawater describes the change of the chemical potential of water per mole of added salt, expressed as multiples of the thermal energy, $R(T_0 + t)$ (Millero and Leung (1976), Feistel and Marion (2007), Feistel (2008)),

$$\mu^W(0,t,p) = \mu^W(S_A,t,p) + m_{SW} R (T_0 + t) \phi. \quad (2.14.2)$$

Here, $R = 8.314 \, 472 \, J \, mol^{-1} \, K^{-1}$ is the universal molar gas constant. The molality $m_{SW}$ is the number of dissolved moles of solutes (ions) of the Reference Composition as defined by Millero et al. (2008a), per kilogram of pure water. Note that the molality of seawater may take different values if neutral molecules of salt rather than ions are counted (see the discussion on page 519 of Feistel and Marion (2007)). The freezing-point lowering equations (3.33.1, 3.33.2) or the vapour-pressure lowering can be computed from the osmotic coefficient of seawater (see Millero and Leung (1976), Bromley et al. (1974)).