Notes on the function gsw_isopycnal_vs_ntp_CT_ratio(SA,CT,p)

This function \texttt{gsw_isopycnal_vs_ntp_CT_ratio}(SA,CT,p) evaluates the “isopycnal temperature gradient ratio” defined by (from section 3.17 of the TEOS-10 Manual, IOC et al. (2010))

\[ G^\theta = \frac{[R_p - 1]}{[R_p/r - 1]} . \]  

This is the ratio of the (parallel) gradient of Conservative Temperature in a potential density surface, \( \nabla_s \Theta \), to that in a neutral tangent plane, \( \nabla_n \Theta \), since, from Eqn. (3.17.3) of the TEOS-10 Manual,

\[ \nabla_s \Theta = r \frac{[R_p - 1]}{[R_p/r - 1]} \nabla_n \Theta = G^\theta \nabla_n \Theta . \]  

This function, \texttt{gsw_isopycnal_vs_ntp_CT_ratio}(SA,CT,p), uses the 75-term polynomial function expression for specific volume \texttt{gsw_specvol}(SA,CT,p). This 75-term polynomial expression for specific volume is discussed in Roquet et al. (2015) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC et al. (2010)). For dynamical oceanography we may take the 75-term polynomial expression for specific volume as essentially reflecting the full accuracy of TEOS-10.

References


Here follows section 3.17 of the TEOS-10 Manual (IOC et al. (2010)).

3.17 Property gradients along potential density surfaces

The two-dimensional gradient of a scalar \( \varphi \) along a potential density surface \( \nabla_s \varphi \) is related to the corresponding gradient in the neutral tangent plane \( \nabla_n \varphi \) by

\[ \nabla_s \varphi = \nabla_n \varphi + \frac{\varphi}{\Theta} \frac{R_p}{R_p - r} \nabla_n \Theta . \]  

(from McDougall (1987a)), where \( r \) is the ratio of the slope on the \( S_\Lambda - \Theta \) diagram of an isoline of potential density with reference pressure \( p_t \) to the slope of a potential density surface with reference pressure \( p \), and is defined by

\[ r = \frac{\alpha^\theta (S_\Lambda, \Theta, p_t)/\beta^\theta (S_\Lambda, \Theta, p_t)}{\alpha^\theta (S_\Lambda, \Theta, p)/\beta^\theta (S_\Lambda, \Theta, p)} . \]
Substituting $\varphi = \Theta$ into (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of $\Theta$

$$\nabla_\sigma \Theta = \left[ \frac{r}{R_p} - \frac{1}{r} \right] \nabla_n \Theta = G^\Theta \nabla_n \Theta$$  \hspace{1cm} (3.17.3)

where the “isopycnal temperature gradient ratio”

$$G^\Theta = \left[ \frac{R_p - 1}{R_p / r - 1} \right]$$  \hspace{1cm} (3.17.4)

has been defined as a shorthand expression for future use. This ratio $G^\Theta$ is available in the GSW Toolbox from the algorithm $\text{gsw_isopycnal_vs_ntp_CT_ratio}$, while the ratio $r$ of Eqn. (3.17.2) is available there as $\text{gsw_isopycnal_slope_ratio}$. Substituting $\varphi = S_A$ into Eqn. (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of $S_A$

$$\nabla_\sigma S_A = \left[ \frac{R_p - 1}{R_p / r - 1} \right] \nabla_n S_A = \frac{G^\Theta}{r} \nabla_n S_A.$$  \hspace{1cm} (3.17.5)