Notes on the function, gsw_entropy_from_CT(SA, CT), which evaluates specific entropy from Conservative Temperature

This function, gsw_entropy_from_CT, finds \( \eta = \eta(S_A, \Theta) \), specific entropy as a function of Absolute Salinity and Conservative Temperature. This is done by first evaluating potential temperature \( \theta \) (with reference pressure \( p_f = 0 \) dbar) from the function gsw_pt_from_CT and then calling the temperature derivative of the Gibbs function as follows,

\[
\eta = \tilde{\eta}(S_A, \theta) = -g_T(S_A, \theta, p=0).
\]  

(1)

Here follows appendix A.10 of the TEOS-10 Manual (IOC et al. (2010)).

A.10 Proof that \( \theta = \theta(S_A, \eta) \) and \( \Theta = \Theta(S_A, \theta) \)

Consider changes occurring at the sea surface, (specifically at \( p = 0 \) dbar) where the temperature is the same as the potential temperature referenced to 0 dbar and the increment of pressure \( dp \) is zero. Regarding specific enthalpy \( h \) and chemical potential \( \mu \) to be functions of entropy \( \eta \) (in place of temperature \( t \)), that is, considering the functional form of \( h \) and \( \mu \) to be \( h = h(S_A, \eta, p) \) and \( \mu = \mu(S_A, \eta, p) \), it follows from the fundamental thermodynamic relation (Eqn. (A.7.1)) that

\[
\begin{align*}
\hat{h}_\eta(S_A, \eta, 0) \, d\eta + \hat{s}_{S_A}(S_A, \eta, 0) \, dS_A &= (T_0 + \theta) d\eta + \mu(S_A, \eta, 0) \, dS_A. \\
\end{align*}
\]  

(A.10.1)

which shows that specific entropy \( \eta \) is simply a function of Absolute Salinity \( S_A \) and potential temperature \( \theta \), that is \( \eta = \eta(S_A, \theta) \), with no separate dependence on pressure. It follows that \( \theta = \theta(S_A, \eta) \).

Similarly, from the definition of potential enthalpy and Conservative Temperature in Eqns. (3.2.1) and (3.3.1), at \( p = 0 \) dbar it can be seen that the fundamental thermodynamic relation (A.7.1) implies

\[
\begin{align*}
c^\theta_p \, d\Theta &= (T_0 + \theta) d\eta + \hat{\mu}(S_A, \theta, 0) \, dS_A. \\
\end{align*}
\]  

(A.10.2)

This shows that Conservative Temperature is also simply a function of Absolute Salinity and potential temperature, \( \Theta = \Theta(S_A, \theta) \), with no separate dependence on pressure. It then follows that \( \Theta \) may also be expressed as a function of only \( S_A \) and \( \eta \). It follows that \( \Theta \) has the “potential” property.