Notes on the function gsw_Turner_Rsubrho(SA, CT, p) which evaluates the Turner angle and the Stability Ratio

This function, gsw_Turner_Rsubrho(SA,CT,p), evaluates the Turner angle Tu and the Stability Ratio \( R_\rho \) of the water column using the 75-term expression, \( \dot{\nu}(S_\Lambda,\Theta,z) \). This 75-term polynomial expression for specific volume is discussed in Roquet et al. (2015) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC et al. (2010)). For dynamical oceanography we may take the 75-term polynomial expression for specific volume as essentially reflecting the full accuracy of TEOS-10.

This function gsw_Turner_Rsubrho(SA,CT,p) evaluates the expressions in Eqns. (3.15.1) and (3.16.1) of the TEOS-10 Manual (IOC et al. (2010)) (see also McDougall et al. (1988)).

References

Here follows sections 3.15 and 3.16 of the TEOS-10 Manual (IOC et al. (2010)).

3.15 Stability ratio

The stability ratio \( R_\rho \) is the ratio of the vertical contribution from Conservative Temperature to that from Absolute Salinity to the static stability \( N^2 \) of the water column. From (3.10.1) above we find

\[
R_\rho = \frac{\alpha_S \Theta}{\beta_T (S_\Lambda)_z}.
\]  
(3.15.1)

The stability ratio \( R_\rho \) is available in the GSW Oceanographic Toolbox from the function gsw_Turner_Rsubrho.
3.16 Turner angle

The Turner angle $Tu$, named after J. Stewart Turner, is defined as the four-quadrant arctangent (Ruddick (1983) and McDougall et al. (1988), particularly their Figure 1)

$$Tu = \tan^{-1}(\alpha^0\Theta_z + \beta^0(S_A)_z, \alpha^0\Theta_z - \beta^0(S_A)_z),$$

where the first of the two arguments of the arctangent function is the "y"-argument and the second one the "x"-argument, this being the common order of these arguments in Fortran and MATLAB. The Turner angle $Tu$ is quoted in degrees of rotation. Turner angles between 45° and 90° represent the “salt-finger” regime of double-diffusive convection, with the strongest activity near 90°. Turner angles between −45° and −90° represent the “diffusive” regime of double-diffusive convection, with the strongest activity near −90°. Turner angles between −45° and 45° represent regions where the stratification is stably stratified in both $\Theta$ and $S_A$. Turner angles greater than 90° or less than −90° characterize a statically unstable water column in which $N^2 < 0$. As a check on the calculation of the Turner angle, note that $R_p = -\tan(Tu + 45°)$. The Turner angle and the stability ratio are available in the GSW Oceanographic Toolbox from the function gsw_Turner_Rsubrho.

The figure below, from McDougall et al. (1988), illustrates the Turner angle on a diagram whose axes should be $(\beta^0(S_A)_z, \alpha^0\Theta_z)$.

References