Notes on the GSW internal library function gsw_Fdelta(p,long,lat)

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The practice in numerical models pre-TEOS-10 has been to treat the model's salinity variable as a perfectly conserved quantity in the interior of the ocean. In order to continue this practice, the appropriate model salinity variable is Preformed Salinity S_*. Preformed Salinity and Absolute Salinity are related to Reference Salinity S_{R} and to S_* respectively by Eqns. (A.20.1) and (A.20.2) of the TEOS-10 Manual, repeated here

$$S_* \;=\; S_{\mathrm{R}}\left(1 - r_1 R^{\delta}\right), \tag{1}$$

$$S_{\mathrm{A}} \;=\; S_*\left(1 + F^{\delta}\right), \tag{2}$$

where

$$R^{\delta} \;\equiv\; \frac{\delta S_{\mathrm{A}}^{\mathrm{atlas}}}{S_{\mathrm{R}}^{\mathrm{atlas}}} \qquad \text{and} \qquad F^{\delta} \;=\; \frac{\left[1+r_1\right]R^{\delta}}{\left(1 - r_1 R^{\delta}\right)} \,. \tag{3a, b}$$

Recall that $R^{\delta} \equiv \delta S_{\mathrm{A}}^{\mathrm{atlas}} \big/ S_{\mathrm{R}}^{\mathrm{atlas}}$ is the ratio of the reference (ref) values of Absolute Salinity Anomaly and Reference Salinity.

The GSW function **gsw_Fdelta**(p,long,lat) finds R^{δ} by calling **gsw_SAAR**(p,long,lat) and then simply calculates F^{δ} from Eqn. (3b) with r_1 being the constant 0.35 based on the work of Pawlowicz *et al.* (2011). Note that since R^{δ} is everywhere less than 0.001 in the global ocean, F^{δ} is little different to $1.35R^{\delta}$. The remainder of this help file explains in a little more detail how F^{δ} is to be used in ocean modelling under TEOS-10.

Because Preformed Salinity S_* is designed to be a conservative salinity variable, blind to the effects of biogeochemical processes, its evolution equation is in the conservative form (see appendix A.21 of IOC *et al.* (2011)),

$$\frac{\mathrm{d}\hat{S}_*}{\mathrm{d}t} \;=\; h^{-1}\nabla_n \cdot \left(hK\nabla_n \hat{S}_*\right) \;+\; \left(D\frac{\partial \hat{S}_*}{\partial z}\right)_z \,. \tag{4}$$

Here the over-tilde of $\hat{S}_*$ indicates that this variable is the thickness-weighted average Preformed Salinity, having been averaged between a pair of closely-spaced neutral tangent planes. The material derivative on the left-hand side of Eqn. (4) is with respect to the sum of the Eulerian and quasi-Stokes velocities of height coordinates (equivalent to the description in appendix A.21 of IOC *et al.* (2010) in terms of the thickness-weighted mean horizontal velocity and the mean dianeutral velocity), while the right-hand side of this equation is the standard notation indicating that $\hat{S}_*$ is being diffused along neutral tangent planes with the diffusivity K and in the vertical direction with the diapycnal diffusivity D (and h here is the average thickness between two closely spaced neutral tangent planes). The model is initialized with values of Preformed Salinity using Eqn. (1) based on observations of Practical Salinity and on the interpolated global observed data base of R^{δ}; this is best done by calling **gsw_Sstar_from_SP**.

In order to evaluate density during the running of an ocean model, Absolute Salinity must be evaluated based on the model's primary salinity variable, Preformed Salinity, and Eqn. (2). This can be done by carrying the following evolution equation for F^{δ}

$$\frac{\mathrm{d}F^{\delta}}{\mathrm{d}t} \;=\; h^{-1}\nabla_n \cdot \left(hK\nabla_n F^{\delta}\right) \;+\; \left(D\frac{\partial F^{\delta}}{\partial z}\right)_z \;+\; \tau^{-1}\left(F^{\delta\,\mathrm{obs}} - F^{\delta}\right). \tag{5}$$

The model variable F^{δ} (note that $F^{\delta} = S_{\mathrm{A}}/S_* - 1$) is initialized based on observations of $R^{\delta} \equiv \delta S_{\mathrm{A}}^{\mathrm{atlas}} \big/ S_{\mathrm{R}}^{\mathrm{atlas}}$ and the use of Eqn. (3b); this is best done by calling **gsw_Fdelta**. Equation (5) shows that F^{δ} is advected and diffused like any other tracer, but in addition, there is a non-conservative source term $\tau^{-1}\left(F^{\delta\,\mathrm{obs}} - F^{\delta}\right)$ which serves to restore the model variable F^{δ} towards the observed value (found from **gsw_Fdelta**) with a restoring time τ that can be chosen to suit particular modeling needs (see the discussion in appendix A.20 of the TEOS-10 Manual, IOC *et al*. (2010)).

In summary, the approach for handling salinity in ocean models suggested in IOC *et al*. (2010) and summarized here carries the evolution Eqns. (4) and (5) for $\hat{S}_{*}$ and F^{δ}, while $\hat{S}_{\mathrm{A}}$ is calculated from these two model variables at each time step according to

$$\hat{S}_{\mathrm{A}} \;=\; \hat{S}_{*}\left(1+F^{\delta}\right). \tag{6}$$

It is this salinity, $\hat{S}_{\mathrm{A}}$, which is used as the argument for the model's expression for density at each time step of the model.

The Baltic Sea is somewhat of an exception because its compositional variations are not due to biogeochemistry but to anomalous riverine input of dissolved salts which behave conservatively. Preformed Salinity S_{*} in the Baltic is equal to Absolute Salinity S_{A}, which implies that $r_1 = -1$ in the Baltic Sea. Hence in the Baltic, an ocean model simply puts $S_{\mathrm{A}} = S_{*}$ and the value of Absolute Salinity Anomaly δS_{A} is immaterial during the running of the model. Of course the values of δS_{A} in the Baltic are important for relating Absolute Salinity and Preformed Salinity to measured values of Practical Salinity there.

If an ocean model is to be run for only a short time (less than a century) then it may be sufficiently accurate to carry only one salinity variable, namely Absolute Salinity S_{A}. For longer integrations the neglect of the non-conservative biogeochemical source term means that the model's salinity variable S_{A} will depart from reality. A more detailed discussion of these points is available in appendix A.20 of the TEOS-10 Manual (IOC *et al*. (2010)).

In summary, the changes needed to make ocean models TEOS-10 compatible are

(i) use an equation of state in terms of S_{A} and Θ, $\hat{\rho}\left(S_{\mathrm{A}},\Theta,p\right)$, such as the 48-term expression to be found in **gsw_rho**(SA,CT,p),

(ii) have Conservative Temperature Θ as the model's temperature variable (note that SST needs to be evaluated in the model's air-sea flux module using **gsw_pt_from_CT** at the sea surface only),

(iii) incorporate the effects of the spatially variable seawater composition using the techniques of appendix A.20 of IOC *et al*. (2010) as summarized above,

(iv) restoring boundary conditions for ocean-only models can be imposed on the model variables S_{*} and Θ,

(v) model output salinities and temperatures are best made as Absolute Salinity S_{A} and Conservative Temperature Θ, consistent with the variables which will be published in oceanographic journals.

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

Pawlowicz, R., D. G. Wright and F. J. Millero, 2011: The effects of biogeochemical processes on oceanic conductivity/salinity/density relationships and the characterization of real seawater. *Ocean Science*, a preliminary version is available at *Ocean Science Discussions*, **7**, 773–836.