Notes on the function, gsw_CT_from_entropy(SA, entropy), which evaluates Conservative Temperature from specific entropy

This function, **gsw_CT_from_entropy**, finds Conservative Temperature for given values of Absolute Salinity and specific entropy, $\Theta = \Theta(S_A, \eta)$. The function amounts to the following simple calls to two other GSW functions,

```
pt = gsw_pt_from_entropy(SA,entropy);
CT = gsw_CT_from_pt(SA,pt);
```

Here follows appendix A.10 of the TEOS-10 Manual (IOC *et al*. (2010)).

A.10 Proof that $\theta = \theta(S_A, \eta)$ and $\Theta = \Theta(S_A, \theta)$

Consider changes occurring at the sea surface, (specifically at $p = 0$ dbar) where the temperature is the same as the potential temperature referenced to 0 dbar and the increment of pressure dp is zero. Regarding specific enthalpy h and chemical potential μ to be functions of entropy η (in place of temperature t), that is, considering the functional form of h and μ to be $h = \hat{h}(S_A, \eta, p)$ and $\mu = \hat{\mu}(S_A, \eta, p)$, it follows from the fundamental thermodynamic relation (Eqn. (A.7.1)) that

$$\hat{h}_\eta(S_A, \eta, 0)\,\mathrm{d}\eta + \hat{h}_{S_A}(S_A, \eta, 0)\,\mathrm{d}S_A = (T_0 + \theta)\mathrm{d}\eta + \mu(S_A, \eta, 0)\,\mathrm{d}S_A, \qquad \text{(A.10.1)}$$

which shows that specific entropy η is simply a function of Absolute Salinity S_A and potential temperature θ, that is $\eta = \eta(S_A, \theta)$, with no separate dependence on pressure. It follows that $\theta = \theta(S_A, \eta)$.

Similarly, from the definition of potential enthalpy and Conservative Temperature in Eqns. (3.2.1) and (3.3.1), at $p = 0$ dbar it can be seen that the fundamental thermodynamic relation (A.7.1) implies

$$c_p^0\,\mathrm{d}\Theta = (T_0 + \theta)\mathrm{d}\eta + \tilde{\mu}(S_A, \theta, 0)\,\mathrm{d}S_A. \qquad \text{(A.10.2)}$$

This shows that Conservative Temperature is also simply a function of Absolute Salinity and potential temperature, $\Theta = \Theta(S_A, \theta)$, with no separate dependence on pressure. It then follows that Θ may also be expressed as a function of only S_A and η. It follows that Θ has the "potential" property.