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A new extended Gibbs thermodynamic potential of seawater

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Abstract

A new and extended Gibbs thermodynamic potential function of seawater is proposed to overcome generally known weaknesses of the International Equation of State of Seawater 1980 and its associated formulas (EOS80). It is valid for applied pressures up to 100 MPa (10,000 dbar), temperatures from −2–40°C, and practical salinities up to 42. At ambient pressure, it is applicable in heat capacity and density up to salinity 50. It includes the triple point of water for reference and is, over its range of validity, fully consistent with the current 1995 international scientific pure water standard, IAPWS95. In conjunction with an improved Gibbs potential of ice, it provides freezing temperatures of seawater for pressures up to 50 MPa (5000 dbar). It is compiled from an extensive set of experimental seawater data, rather than being derived from EOS80 equations. Older seawater data were specifically recalibrated for compatibility with the recent pure water standard. By this procedure, experimental high-pressure densities proved consistent with sound speeds confirmed by deep-sea travel time measurements. Temperatures of maximum density are described within their experimental uncertainty. For very low salinities, Debye-Hückel limiting laws are recompiled using latest physical and chemical constants. The potential function is expressed in the 1990 International Temperature Scale ITS-90.

Contents

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1. Introduction

At present, the most important international standard formulas for seawater are those described in the collection of nine FORTRAN algorithms by Fofonoff & Millard (1983). Two of them are devoted to the relation between electrical conductivity of seawater and the Practical Salinity Scale 1978 (PSS-78, Unesco, 1981b). Both will not be discussed later because conductivity is not an equilibrium property of electrolytic solutions.

Three other formulas cover basic equilibrium properties of seawater as functions of practical salinity, S, Celsius temperature, t, expressed in IPTS-68 scale (Barber, 1969), and applied pressure, p:

- Density of seawater, $\rho(S,t,p)$, representing the International Equation of State of Seawater 1980 (Unesco, 1981c), termed in short EOS80, asssembled by Millero, Chen, Bradshaw & Schleicher (1980) and Millero & Poisson (1981a).
- Specific heat capacity of seawater at constant pressure, $c_P(S,t,p)$, as proposed by Millero, Perron & Desnoyers (1973a).
- Sound speed of seawater, $U(S,t,p)$, after the formula of Chen & Millero (1977).

We shall refer to this triple of functions later jointly as ‘EOS80’ for convenience.

Three more algorithms refer to properties important under the influence of gravity, hydrostatic pressure-depth conversion, adiabatic lapse rate, and potential temperature. They will not further be discussed in the current paper. All of them could in principle have been derived from the EOS80 set, but in fact the latter two are based on the formulas of Bryden (1973).

Finally, a formula for the freezing point temperature of seawater, $t_f(S,p)$, is defined as specified by Millero (1978), and recommended by the Joint Panel on Oceanographic Tables and Standards (Unesco, 1981a). It additionally exploits certain properties of ice (Millero & Leung, 1976b).

Several weaknesses of the well-established EOS80 and associated equations have been revealed over the two decades of their successful use in oceanography. They can be summarized under four aspects:

1. Agreement with experiments

(i) EOS80 does not properly describe high-pressure sound speed as derived from deep-sea travel times (Spiesberger & Metzger, 1991, Dushaw, Worcester, Cornuelle & Howe, 1993, Millero & Li, 1994, Meinen & Watts, 1997)
(ii) Due to (i), there is evidently a pending potential conflict between abyssal travel-time measurements and EOS80 high-pressure densities, which are considered consistent with EOS80 sound speed.
(iii) EOS80 does not properly describe temperatures of maximum density determined experimentally, especially for brackish waters under pressure (Caldwell, 1978, Siedler & Peters, 1986)

2. Consistency with international standards
(iv) EOS80 is not expressed in terms of the international temperature scale ITS-90 (Blanke, 1989, Preston-Thomas, 1990, Saunders, 1990)
(v) At zero salinity, EOS80 shows systematic deviations from the international pure water standard IAPWS95 (Wagner & Pruß, 2002), especially in compressibility, thermal expansion, and sound speed (sections 2-6 of this paper)
(vi) EOS80 is derived from seawater measurements relative to or calibrated with fresh water properties which are partly obsolete with respect the new pure water standard IAPWS95
(vii) EOS80 range of validity does not include the triple point of water which is a standard reference point for thermodynamic descriptions of water

3. Internal consistency
(viii) EOS80 is redundant and contradictory, as certain thermodynamic properties like heat capacity can by computed by combining other equations of EOS80, sometimes leading to very different results, especially near the density maximum
(ix) EOS80 obeys thermodynamic cross-relations (Maxwell relations) only approximately but not identically
(x) Freezing point temperatures are valid for air-saturated water, while other EOS80 formulas are defined for air-free water, thus causing systematic offsets

4. Completeness
(xi) EOS80 does not provide specific enthalpy which is required for the hydrodynamic energy balance by means of the enthalpy flux (Landau & Lifschitz, 1974, Bacon & Fofonoff, 1996, Warren, 1999) or the Bernoulli function (Gill, 1982, Saunders, 1995). Specific enthalpy is further necessary for the calculation of mixing heat (Fofonoff, 1962) or of conservative quantities like potential enthalpy (McDougall, 2003)
(xii) EOS80 does not provide specific entropy as an unambiguous alternative for potential temperature (Feistel & Hagen, 1994). It allows e.g. for an effective and accurate computation of potential temperature and potential density (Bradshaw, 1978, Feistel, 1993), especially in numerical ocean models (McDougall, Jackett, Wright & Feistel, 2003)
(xiii) EOS80 does not provide specific internal energy, which like enthalpy is required for proper energy balances, and e.g. elucidates the changing thermal water and seawater properties when being compressed
(xiv) EOS80 does not provide chemical potentials which allow the computation of properties of vapour pressure or osmotic pressure (Millero and Leung, 1976b), or properties of sea ice (Feistel & Hagen, 1998), or as indicators for oceanic actively mixing layers (Feistel and Hagen, 1994)
(xv) EOS80 density was only later extended to salinity $S = 50$ at ambient pressure by another separate high-salinity equation of state (Poisson, Gadhoumi & Morcos, 1991)
(xvi) Existing freezing point temperatures are valid up to pressures of 5 MPa (500 dbar), which is insufficient for extreme polar systems like Lake Vostok (Siegert, Ellis-Evans, Tranter, Mayer, Petits, Salamantin, et al., 2001)

The thermodynamic potential proposed in the current paper, referred to as ‘F03’ later, overcomes all the problems mentioned above. In the last case, (xvi), it is to be used in conjunction with a compatible Gibbs potential of ice, as specified in section 13.

A proper potential function constitutes a very compact, practical, mathematically elegant and physically consistent way of representing and computing all thermodynamic equilibrium properties of a given substance. Many books and articles treat various aspects of this classical method, like e.g. Fofonoff (1962), Landau & Lifschitz (1966, 1974), Falkenhagen & Ebeling (1971), Tillner-Roth (1998), or, quite recently, the comprehensive review by Alberty (2001).

For pure water, thermodynamic potential functions were quantitatively defined as a standard as early as 1968 by an International Formulation Committee, leading finally to IAPWS95, the current “IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use” (IAPWS, 1996, Wagner and Pruß, 2002). Despite a similar proposal for seawater made by Fofonoff (1962), and even though all necessary additional ingredients for its construction were available at the time when EOS80 was specified (Millero and Leung, 1976b, Millero, 1982, 1983), it took another decade until a first version was assembled, called here ‘F93’ (Feistel, 1993) and improved later, referred to as ‘FH95’ (Feistel and Hagen, 1995). The recently published equation of state of McDougall et al. (2003) is a computationally faster but numerically equivalent formulation of FH95 for use in numerical models, restricted to naturally occuring combinations of salinity, temperature and pressure, so-called ‘Neptunian’ waters, and formulated in terms of potential density as function of salinity, potential temperature, and pressure.

Specific free enthalpy (also called Gibbs function, Gibbs energy, Gibbs free energy, or free energy in the literature) of seawater, $g(S,t,p)$, is assumed to be a polynomial-like function of the independent variables practical salinity (PSS-78) (Lewis & Perkin, 1981, Unesco, 1981b), $S = 40{\cdot}x^2$, temperature (ITS-90) (Blanke, 1989, Preston-Thomas, 1990), $t = 40°\mathrm{C}{\cdot}y$, and applied pressure, $p = 100\ \mathrm{MPa}{\cdot}z$, as,

$$g(S,t,p) = 1\ \mathrm{J/kg}{\cdot}\sum_{j,k}\left\{g_{0jk} + g_{1jk}x^2\ln x + \sum_{i>1} g_{ijk}x^i\right\}y^j z^k \tag{1}$$

We shall use capital symbols $T = T_0 + t$ for absolute temperatures, with $T_0 = 273.15$ K, and $P = P_0 + p$ for absolute pressures, with $P_0 = 0.101325$ MPa, in the following text. IAPWS95 formulas are expressed in density, absolute temperature or absolute pressure as independent variables, and will later be written this way if used in mathematical expressions. The logarithmic term in Eq. (1) follows from Planck’s theory of ideal solutions; it is a pressure-independent linear function of temperature, as shown in Table 1 (compare Eq. (47)). The series expansion with respect to the salinity root results from the Debye-Hückel theory of dilute electrolytes. Both are required for correct behaviour in the limit of infinite dilution (Landau and Lifschitz, 1966, Falkenhagen and Ebeling, 1971).

The structure of the right-hand side of Eq. (1) is convenient for the analytical execution and numerical implementation of partial derivatives by suitable index shifting in the array of coefficients. These derivatives serve for the compilation of all thermodynamic properties in terms of the potential function, Eq. (1), either in explicit form, or by Newton iteration of implicit expressions like those in Eqs. (16)–(21). We list the most relevant oceanographic quantities here for easy reference, as being expressed by the potential function, $g(S,t,p)$, and its partial derivatives.

There are three first derivatives of g with respect to its independent variables p, t, and S.

Density, ρ, density anomaly, γ (also called σ_t), and specific volume, v:

$$\frac{1}{\rho} = \frac{1}{1000\ \mathrm{kg/m^3} + \gamma} = v = \left(\frac{\partial g}{\partial p}\right)_{S,t} \tag{2}$$

Specific entropy, σ:

Table 1
Coefficients of the free enthalpy polynomial being determined in this paper. Variables x, y, z represent salinity root, temperature and pressure in dimensionless form. Numbers in the cells are the sections where the computation of the particular coefficient is described

g_{ijk}	x^0	$x^2 \ln x$	x^2	x^3	x^4	x^5	x^6	x^7
$y^0 z^0$	6	12	12	12	13,14	13,14	13,14	13,14
$y^1 z^0$	6	12	12	12	13,14	13,14	13,14	13,14
$y^2 z^0$	2		7–11	7–11				
$y^3 z^0$	2		7–11	7–11				
$y^4 z^0$	2		7–11	7–11				
$y^5 z^0$	2		7–11					
$y^6 z^0$	2							
$y^7 z^0$	2							
$y^0 z^1$	3–5		7–11	7–11	7–11	7–11		
$y^1 z^1$	3–5		7–11	7–11	7–11			
$y^2 z^1$	3–5		7–11	7–11				
$y^3 z^1$	3–5		7–11	7–11				
$y^4 z^1$	3–5		7–11	7–11				
$y^5 z^1$	3–5							
$y^6 z^1$	3–5							
$y^7 z^1$	3–5							
$y^0 z^2$	3–5		7–11	7–11	7–11			
$y^1 z^2$	3–5		7–11	7–11				
$y^2 z^2$	3–5		7–11	7–11				
$y^3 z^2$	3–5		7–11					
$y^4 z^2$	3–5		7–11					
$y^5 z^2$	3–5							
$y^6 z^2$	3–5							
$y^0 z^3$	3–5		7–11	7–11	7–11			
$y^1 z^3$	3–5		7–11	7–11				
$y^2 z^3$	3–5		7–11	7–11				
$y^3 z^3$	3–5		7–11					
$y^4 z^3$	3–5		7–11					
$y^5 z^3$	3–5							
$y^6 z^3$	3–5							
$y^0 z^4$	3–5		7–11	7–11				
$y^1 z^4$	3–5		7–11					
$y^2 z^4$	3–5		7–11					
$y^3 z^4$	3–5		7–11					
$y^4 z^4$	3–5							
$y^5 z^4$	3–5							
$y^0 z^5$	3–5		7–11					
$y^1 z^5$	3–5		7–11					
$y^2 z^5$	3–5		7–11					
$y^3 z^5$	3–5							
$y^0 z^6$	3–5							

$$\sigma = -\left(\frac{\partial g}{\partial t}\right)_{S,p} \tag{3}$$

Relative chemical potential, μ:

$$\mu = \left(\frac{\partial g}{\partial S}\right)_{t,p} \tag{4}$$

Note that the density anomaly (Eqs. (2), (17)) was originally defined by the Knudsen parameter, $\rho/\rho_{\max}-1$, relative to the maximum density of pure water at atmospheric pressure, $\rho_{\max}$, and, $\gamma = \rho - 1000\ \mathrm{kg/m^3}$, was proposed later as its more practical substitute (Unesco, 1985, 1986, Mamayev, Dooley, Millard, Taira & Morcos, 1991). But, the symbol γ was not accepted by the oceanographic community, and therefore the traditional letter was redefined as, $\sigma = \rho - 1000\ \mathrm{kg/m^3}$ (Siedler, 1998).

Several thermodynamic coefficients require second derivatives of g.

Isothermal compressibility, K:

$$K = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_{S,t} = -\frac{(\partial^2 g/\partial p^2)_{S,t}}{(\partial g/\partial p)_{S,t}} \tag{5}$$

Isobaric thermal expansion coefficient, α:

$$\alpha = \frac{1}{v}\left(\frac{\partial v}{\partial t}\right)_{S,p} = \frac{(\partial^2 g/\partial t\partial p)_S}{(\partial g/\partial p)_{S,t}} \tag{6}$$

Isobaric specific heat capacity, c_P:

$$c_P = T\left(\frac{\partial \sigma}{\partial t}\right)_{S,p} = -T\left(\frac{\partial^2 g}{\partial t^2}\right)_{S,p} \tag{7}$$

Isothermal haline contraction coefficient, β:

$$\beta = -\frac{1}{v}\left(\frac{\partial v}{\partial S}\right)_{t,p} = -\frac{(\partial^2 g/\partial p\partial S)_t}{(\partial g/\partial p)_{S,t}} \tag{8}$$

Adiabatic compressibility, κ, and sound speed, U:

$$\kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_{S,\sigma} = \frac{v}{U^2} = \frac{(\partial^2 g/\partial t\partial p)_S^2 - (\partial^2 g/\partial t^2)_{S,p}(\partial^2 g/\partial p^2)_{S,t}}{(\partial g/\partial p)_{S,t}(\partial^2 g/\partial t^2)_{S,p}} \tag{9}$$

Adiabatic lapse rate, Γ:

$$\Gamma = \left(\frac{\partial t}{\partial p}\right)_{S,\sigma} = -\frac{(\partial^2 g/\partial t\partial p)_S}{(\partial^2 g/\partial t^2)_{S,p}} \tag{10}$$

Adiabatic haline contraction coefficient, β':

$$\beta' = -\frac{1}{v}\left(\frac{\partial v}{\partial S}\right)_{\sigma,p} = \frac{(\partial^2 g/\partial S\partial t)_p(\partial^2 g/\partial t\partial p)_S - (\partial^2 g/\partial t^2)_{S,p}(\partial^2 g/\partial S\partial p)_t}{(\partial g/\partial p)_{S,t}(\partial^2 g/\partial t^2)_{S,p}} \tag{11}$$

Further thermodynamic functions are defined by mathematical Legendre transforms (Alberty, 2001).

Specific free energy (also called Helmholtz energy or Helmholtz free energy), f:

$$f = g - Pv = g - P\cdot\left(\frac{\partial g}{\partial p}\right)_{S,t} \tag{12}$$

Specific enthalpy, h:

$$h = g + T\sigma = g - T\cdot\left(\frac{\partial g}{\partial t}\right)_{S,p} \tag{13}$$

Specific internal energy, e:

$$e = g + T\sigma - Pv = g - T\cdot\left(\frac{\partial g}{\partial t}\right)_{S,p} - P\cdot\left(\frac{\partial g}{\partial p}\right)_{S,t} \tag{14}$$

Chemical potential of water in seawater, μ^W:

$$\mu^W = g - S\mu = g - S\cdot\left(\frac{\partial g}{\partial S}\right)_{t,p} \tag{15}$$

Oceanographic so-called 'potential' quantities can be obtained by formally replacing in-situ temperature t and in-situ pressure p by potential temperature θ and reference pressure p_r. They describe the property a water parcel would take if moved from in-situ pressure p to reference pressure p_r without exchange of matter and heat. Due to the latter, by definition of θ, specific entropy is equal to 'potential' specific entropy.

Potential temperature, $\theta(S,t,p,p_r)$, is implicitly given by (Bradshaw, 1978, Feistel, 1993):

$$\sigma(S,t,p) = \sigma(S,\theta,p_r) \tag{16}$$

Potential density, ρ_θ, and potential density anomaly, γ_θ (also called σ_θ):

$$\rho_\theta(S,t,p,p_r) = 1000\ \mathrm{kg/m^3} + \gamma_\theta = \rho(S,\theta(S,t,p,p_r),p_r) \tag{17}$$

Potential specific enthalpy, h_θ (McDougall, 2003):

$$h_\theta(S,t,p,p_r) = h(S,\theta(S,t,p,p_r),p_r) \tag{18}$$

Equilibria between seawater and other aqueous phases are controlled by equal chemical potentials of water in both.

Osmotic pressure, $\pi(S,t,p)$, is implicitly given by:

$$\mu^W(S,t,p+\pi) = \mu^W(0,t,p) \tag{19}$$

Freezing point temperature, $t_f(S,p)$, is implicitly given by (requiring additionally free enthalpy of water ice, g^{Ice}, see section 13):

$$\mu^W(S,t_f,p) = g^{Ice}(t_f,p) \tag{20}$$

Vapour pressure, $p_V(S,t,p)$, above seawater at pressure p, is implicitly given by (requiring additionally free enthalpy of water vapour, g^{Vapour}, as available by IAPWS95):

$$\mu^W(S,t,p) = g^{Vapour}(t,p_V) \tag{21}$$

The functions defined by Eqs. (12)–(15) can serve as alternative thermodynamic potentials if formulated in terms of their particular natural independent variables (Alberty, 2001). This may lead to more compact and effective expressions, like e.g. the use of enthalpy (13) as a function of salinity, entropy, and pressure for the description of adiabatic processes, but it may require e.g. an additional temperature-entropy conversion formula because entropy cannot be directly measured experimentally (Feistel and Hagen, 1995, Tillner-Roth, 1998, McDougall et al., 2003).

A Gibbs potential of sea ice is immediately available from a suitable combination of the Gibbs potentials of seawater and of ice (Feistel and Hagen, 1998). It provides all equilibrium properties of sea ice like e.g. brine salinity, isochoric pressure coefficient, or its exceptionally large heat capacity, which results from the addition of the latent and dilution heats of internal phase transition processes to the heat capacities of brine and ice.

The physico-chemical properties of seawater listed above are sometimes distinguished between so-called 'PVT' and 'thermochemical' properties (Millero, 1982, 1983). The first is an abbreviation for 'Pressure-Volume-Temperature' and comprises quantities mostly related to mechanical forces and dimensions like

density and sound speed, as well as their combinations or derivatives. They are available from measurements, do not contain freely adjustable constants, and do not possess the logarithm term of Eq. (1). The second group of properties deals with energy, entropy, enthalpy, and chemical potentials, as well as quantities derived thereof. In many cases, only their differences, but not the quantities themselves are available from experiments. Therefore they can depend on freely adjustable gauge constants like the absolute energy and absolute entropy of water and of every single dissolved component. The so-called 'colligative' properties like freezing point depression or osmotic pressure of a solution are assigned to the thermochemical group. This classification is not unique; heat capacity is often counted in the second category for obvious reasons, but it can be expressed by density and sound speed using thermodynamic rules. Therefore, we consider here the EOS80 triple as a complete description of seawater PVT properties, and the related freezing point formula as its incomplete thermochemical counterpart.

The four free constants of F03 are defined, as opposed to FH95 and F93, by the conditions of vanishing internal energy and entropy of the liquid phase at the triple point of water at $S = 0$, $T_t = 273.16$ K, $P_t = 611.657$ Pa, and vanishing enthalpy and entropy of standard ocean seawater at $S = 35$, $T = T_0 = 273.15$ K, $P = P_0 = 101325$ Pa.

The equilibrium properties of seawater depicted by the actual Gibbs potential are, in the strict sense, only valid for gravity-free conditions. Seawater exposed to a centrifugal or gravity force is at thermodynamic equilibrium if temperature and all chemical potentials are constant throughout the volume. Under such circumstances other quantities like pressure or entropy, however, will exhibit spatial gradients proportional to the external field (Landau and Lifschitz, 1966, 1974, Alberty, 2001). The chemical potentials of the various components forming seasalt depend on pressure in different ways, consequently, the equilibrium stoichiometric composition of seasalt will vary with the pressure level in the water column. Practically this is not observed in the ocean because the relaxation time towards this equilibrium significantly exceeds time scales of oceanic processes, especially turbulent mixing, as well as reasonable observation periods. Molecular baro-diffusion is slow and terrestrial gravity is weak enough to make gravity-free thermodynamics the much better approximation for natural oceanographic conditions, thus allowing for a greatly simplified description with formally only seasalt as single solute (Fofonoff, 1962, 1985).

Variations in the chemical composition of seasalt are observed in regions suffering from reduced water exchange with the world ocean, like in the North Sea (Krümmel, 1893), the Baltic Sea (Rohde, 1966), the Bering Sea (Tsunogai, Kusakabe, Iizumi, Koike & Hattori, 1979), or the Red Sea (Poisson, Lebel & Brunet, 1981). In such cases, true densities may deviate up to 0.1 kg/m^3 from those formally derived by means of conductivity measurements (Brewer & Bradshaw, 1975, Millero & Kremling, 1976c, Millero & Sohn, 1992, Feistel, 1998, Millero, 2000). Their impact on ocean dynamics is usually only small because the anomalies vary smoothly in space, or are masked by strong absolute density gradients (McDougall et al., 2003). Composition anomalies will not further be considered in this paper.

It is obvious from Eqs. (2)–(21) that a particular coefficient of the thermodynamic potential function portrays itself through various different thermodynamic quantities. Vice versa, any kind of measured equilibrium property can be included into the determination of the potential function from experiments. The method used here for this purpose, multi-variable linear and nonlinear regression, has to achieve suitable compromises between the overlapping requirements posed by the different data and their particular scatter. For this goal, we have systematically specified numerical weights by estimated or reported experimental errors, called 'required r.m.s.' in the further text, where r.m.s. stands for root mean square deviation. The abbreviation ppm denotes ratios in parts per million (1/1,000,000).

The computation of coefficients belonging to the pure water part is described in sections 2 to 6 of this paper, compare Table 1. One fit was carried out for heat capacity at ambient pressure (section 2), another one for all high-pressure properties (sections 3 to 5), and two coefficients are implied by the reference state definition (section 6). A single regression was run for all PVT properties of seawater (sections 7 to 11), another one for thermochemical properties (sections 13, 14), after the required Debye-Hückel limiting

laws had been evaluated in section 12. For details of the regression algorithm we refer to the former papers about F93 and FH95.

2. Heat capacity of water

Isobaric specific heat capacity, c_P, of water in EOS80 was taken from the paper of Millero et al. (1973a), who used as pure water reference the data of de Haas, composed of several measurements between 1902 and 1927, published by Stimson (1955). FH95 was computed from exactly the same function. The accuracy of these data is not given, but is likely in the range of several J/(kg K), as suggested by Millero et al. (1973a), who found a deviation of 2 J/(kg K) from the earlier measurements of Cox & Smith (1959), Bromley (1968) and Bromley, Diamond, Salami & Wilkins (1970). The precision of seawater measurements relative to pure water is, however, much better with their typical deviations up to only 0.5 J/(kg K) (Millero et al., 1973a).

IAPWS95 heat capacities deviate up to 6 J/(kg K) from the recent measurements of Archer & Carter (2000), thus obeying the experimental accuracy of 13 J/(kg K) of the latter, see Fig. 1. EOS80 and FH95 heat capacities are systematically lower by about 2 J/(kg K). Data reported by Bromley et al. (1970), maximum error given there is 2.2 cal/(kg K), converted 9 J/(kg K), are in very good agreement with EOS80 and FH95 after transforming temperatures from IPTS-48 to ITS-90 and calories to Joule by 4.1840 J/cal. They are a smoothed version of seawater measurements published by Bromley, Desaussure, Clipp & Wright (1967), based on pure water data of Osborne, Stimson & Ginnings (1939).

New coefficients g_{0j0}, j > 1 have been determined by fitting the least square integral

$$\int \left\{ T\left(\frac{\partial^2 g}{\partial t^2}\right)_{S=0,p=0} + c_P^{IAPWS95}(T,P_0) \right\}^2 dt = Min \tag{22}$$

with respect to IAPWS95 heat capacities at atmospheric pressure between the freezing point and 45 °C, as given in the appendix. The r.m.s. of the fit was 0.01 J/(kg K), which is well below the experimental uncertainty. High pressure heat capacities computed after performing the density regression as described in sections 3–5, by $c_p = -T\left(\frac{\partial^2 g}{\partial t^2}\right)_{S=0,p}$, show a stronger deviation of up to 0.1 J/(kg K) at high temperatures, see Fig. 2.

The estimated experimental uncertainty Δc_P of about 10 J/(kg K) even in latest measurements of heat capacities is remarkably high and significantly exceeds the scatter between different accurate experiments. This value does not only apply to pure water, but to seawater as well since the most reliable measurements of the latter (Millero et al., 1973a) precisely detected their readings only relative to a pure water reference. We can estimate the possible influence of changes Δc_P on other important properties.

Uncertainties in the determination of the adiabatic lapse rate $\Gamma = \left(\frac{\partial t}{\partial p}\right)_{S,\sigma} = \frac{\alpha T v}{c_P}$ (Fofonoff and Millard, 1983) are given by,

$$\frac{\Delta\Gamma}{\Gamma} = \frac{\Delta\alpha}{\alpha} + \frac{\Delta v}{v} - \frac{\Delta c_P}{c_P} \tag{23}$$

Typical seawater values are e.g. $\Delta\alpha \approx 0.6$ ppm/K, $\alpha = 100$ ppm/K, $\Delta v/v \approx 1$ ppm, $\Delta c_P \approx 10$ J/(kg K), $c_P = 4000$ J/(kg K), $\Gamma = 10$ mK/MPa. Here, $\Delta\alpha/\alpha \approx 0.6\%$and $\Delta c_P/c_P \approx 0.3\%$ contribute the biggest shares to the total error. Reported experimental accuracies of lapse rates $\Delta\Gamma/\Gamma \approx 0.4\%$(Rögener & Soll, 1980, Wagner and Pruß, 2002) or $\Delta\Gamma/\Gamma \approx 0.2\%$ (Caldwell & Eide, 1980) are of similar size. These estimates agree quite

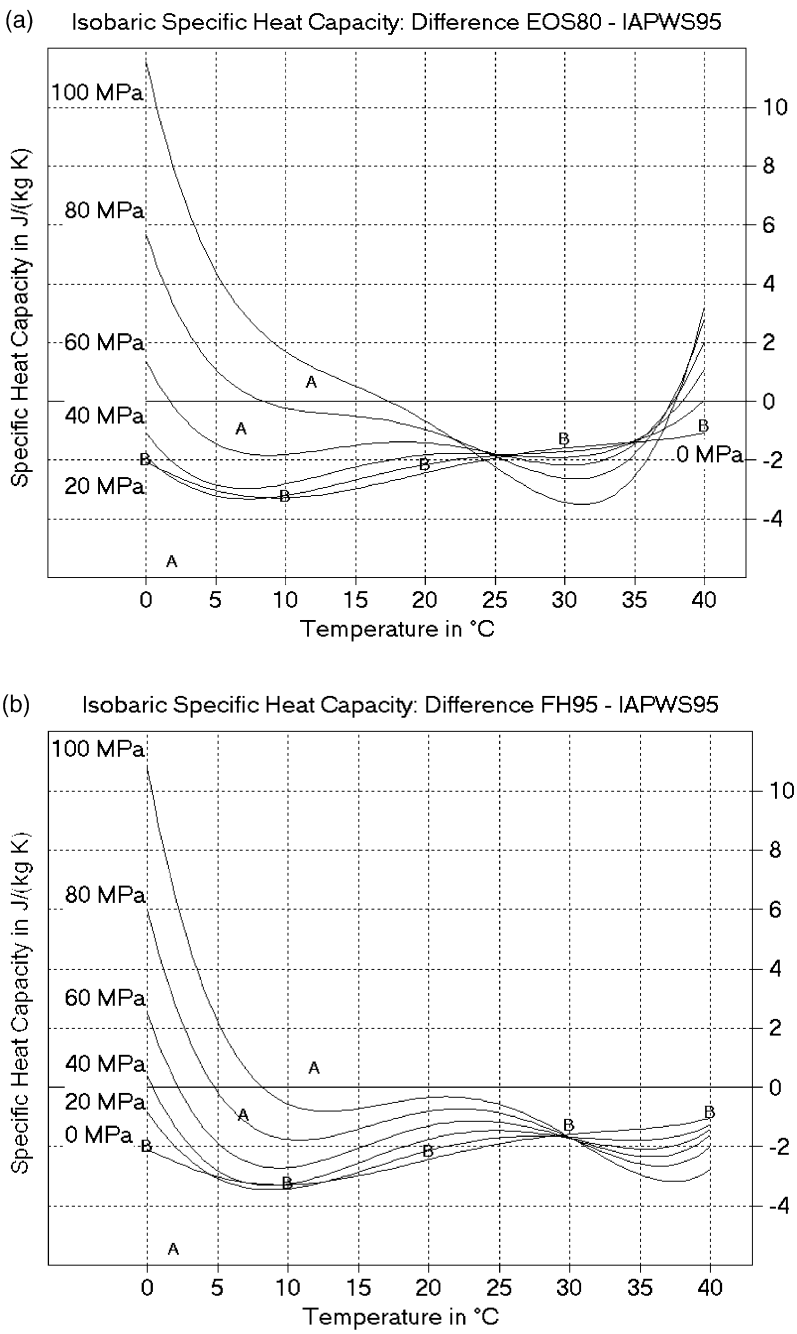

Fig. 1. Deviation of a) EOS80, above, and b) FH95, below, isobaric specific heat capacities, $c_P(0,t,p)$, from IAPWS95 at applied pressures p between 0 MPa and 100 MPa. Measurements at atmospheric pressure (P = 0.101325 MPa) of Archer and Carter (2000) with accuracy 13 J/(kg K) are shown as points "A", those reported by Bromley et al. (1970) with accuracy 9 J/(kg K), as "B", taken originally from Osborne et al. (1939), with precision 100 ppm.

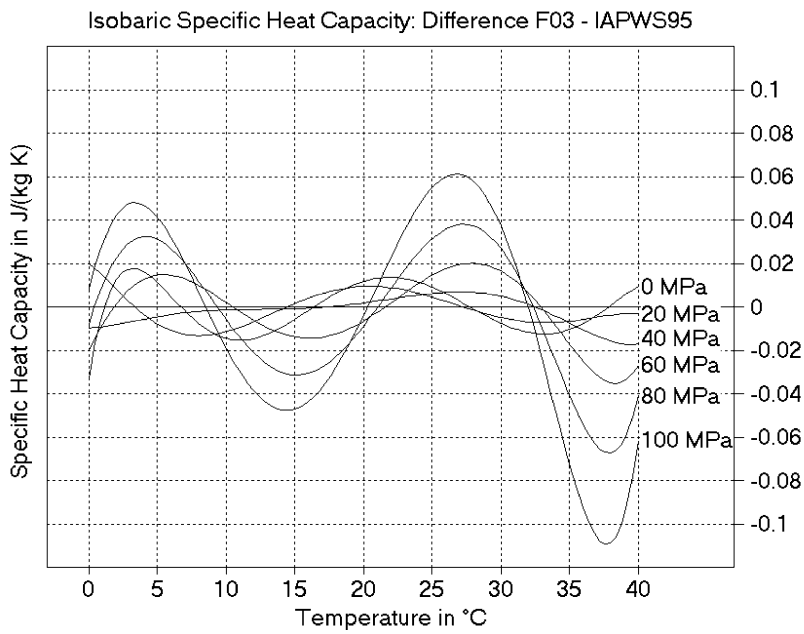

Fig. 2. Deviation between isobaric heat capacity, $c_P(0,t,p)$, of F03 from IAPWS95 at various applied pressures between 0 and 100 MPa (10,000 dbar). Note that only data at atmospheric pressure were used for the fit.

well with a potential temperature error of 6 mK over a pressure change of 100 MPa (Fofonoff and Millard, 1983).

3. Thermal expansion of water

The isobaric thermal expansion coefficient of water α can be expressed in terms of temperature t and pressure p by first and second derivatives of free enthalpy g, as given by Eq. (6). Due to the density anomaly of water, α becomes negative below the temperature of maximum density, t_{MD}. By definition, thermal expansion is theoretically zero at the t_{MD} points determined experimentally by Caldwell (1978). From his accuracy of $\Delta t = 40$ mK follows $\left|\Delta\frac{\partial v}{\partial t}\right| \approx \left|\frac{\partial^2 v}{\partial t^2}\right| \Delta t \approx 0.6$ mm^3/(kg K) or $|\Delta\alpha| \approx 0.6$ ppm/K at the measured t_{MD} (Feistel and Hagen, 1995). At other temperatures, this error range may be considered as a quality criterion of density formulas in general (McDougall et al., 2003). This value is in good agreement with the deviation of 0.4 ppm/K (Millero, Chen, Bradshaw & Schleicher, 1981c) between the thermal expansion coefficients of Bigg (1967) and Kell (1975), and with the measurements of Bradshaw & Schleicher (1970).

In the IAPWS95 formulation, the thermal expansion coefficient is expressed as,

$$\alpha(T,\rho) = \frac{1(\partial P/\partial T)_\rho}{\rho(\partial P/\partial \rho)_T} \tag{24}$$

with $P(T,\rho) = \rho^2\left(\frac{\partial f}{\partial \rho}\right)_T$ and its inverse "backward" function provided, $\rho = \rho(T,P)$.

The deviations of thermal expansion coefficients of EOS80 and FH95 from IAPWS95 are shown in Fig.

3(a) and (b). Only at pressures below 20 MPa do the residuals remain within the desired error range. The strongest differences appear at low temperatures and high pressures and exceed this range by more than a factor of 10.

The coefficients g_{0jk}, $j > 0$, $k > 0$, have been determined by a least-square fit of

$$\int\left\{\left(\frac{\partial^2 g}{\partial t\partial p}\right)_{S=0}-\frac{1}{\rho^2}\frac{(\partial P/\partial T)_\rho}{(\partial P/\partial\rho)_T}\right\}^2 dpdt = Min \tag{25}$$

with respect to the functions $\rho(T,P)$ and $P(T,\rho)$ provided by IAPWS95, together with the fits of density (section 4) and compressibility (section 5). Thermal expansion was treated in two data groups, the first group contains values at $p = 0$ MPa and temperatures between the freezing point and 45 °C with a required r.m.s. deviation of 0.6 mm^3/(kg K), the second at pressures between triple point and 110 MPa and temperatures between the pressure-dependent freezing points and 45 °C with the same r.m.s. Note that $t = 0$ °C at atmospheric pressure is below the freezing temperature of air-free water ($t_f = 0.0026$ °C, Wagner and Pruß, 2002) and is beyond validity of IAPWS95, see also section 13 of this paper. The resulting r.m.s. deviation was 0.005 mm^3/(kg K) for the first and 0.010 mm^3/(kg K) for the second group, as shown in Fig. 4, which is significantly better than the required precision. Determined coefficients are listed in the appendix.

At Caldwell's t_{MD} points, thermal expansion coefficients at various pressures are plotted in Fig. 5, computed with either EOS80 (A), FH95 (B), or IAPWS95 (C). Data of F03 are virtually identical with IAPWS95 in this plot. Above about 3.5 °C (or 2 MPa), IAPWS95 is almost identical with FH95 and slightly higher that EOS80. From 3.5–2 °C (10 MPa), IAPWS95 is in between FH95 and EOS80. From then on, IAPWS95 values are systematically lower than the other two and even exceed the range of 0.6 ppm/K. The comprehensive and consistent description of IAPWS95, however, suggests its higher reliability than Caldwell's measurements in this comparison. Data below the freezing point, where IAPWS95 is not valid (Wagner and Pruß, 2002), are shown in brackets. Despite these details, all four data sets remain on average within the error limit with r.m.s. deviations of 0.55 ppm/K (EOS80), 0.52 ppm/K (FH95) and 0.63 ppm/K (IAPWS95, F03).

The currently best experimental value for the t_{MD} at $p = 0$ MPa is $t_{MD} = 3.983\ 035 \pm 0.000\ 67$ °C (Tanaka, Girard, Davis, Peuto & Bignell, 2001, Wagner and Pruß, 2002), while Caldwell (1978) had determined $t_{MD} = 4.02 \pm 0.04$ °C. The t_{MD} values computed as zeroes of the corresponding thermal expansion formulas are given in Table 2, all are about 2–5 mK lower than Tanaka's et al. t_{MD}. This corresponds to an error in thermal expansion of 0.03–0.08 ppm/K.

4. Density of water

The density of air-free water at atmospheric pressure is described very accurately by EOS80, based on the equation of Bigg (Bigg, 1967, Millero and Poisson, 1981a) within an error limit of $\Delta\rho/\rho \approx 3$ ppm to 6 ppm (Wagenbreth & Blanke, 1971), and consequently by FH95 as well. Near 20 °C, it deviates about 2 ppm from the IAPWS95 standard and 1.5 ppm from the currently recommended density formula of Tanaka et al. (2001) with accuracy better than 1 ppm, see Fig. 6. Thus, the recent measurements are within the error range of EOS80, but EOS80 is not entirely in the range of these measurements.

At higher pressures, densities of Kell & Whalley (1975) with accuracy about 20 ppm are considered the best (Wagner and Pruß, 2002) and have served as reference data for IAPWS95 in the region $t = 0$–50°C, $p = 0$–100 MPa. The pure water part of EOS80, however, is based on the work of Millero et al. (1980, 1981c) with claimed accuracy of 4.3 ppm. The deviation between EOS80 and IAPWS95 is less than 10 ppm at temperatures above 25 °C but exceeds 20 ppm between 0 and 17 °C, see Fig. 7. Thus,

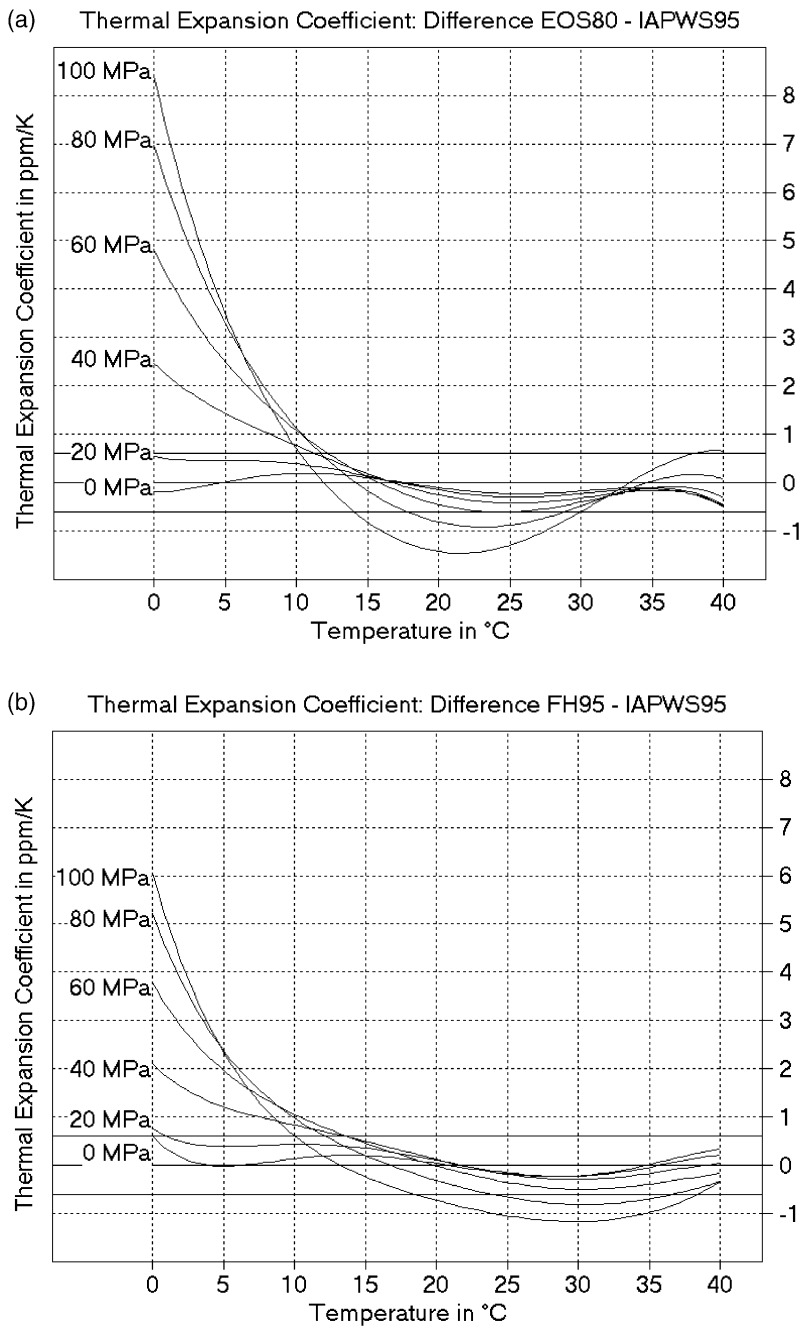

Fig. 3. Comparison of isobaric thermal expansion coefficients, $\alpha(0,t,p)$, of (a) EOS80, above and (b) FH95, below, with IAPWS95 at applied pressures p between 0 and 100 MPa (0 and 10,000 dbar). The anticipated error range of 0.6 ppm/K is shown by solid lines.

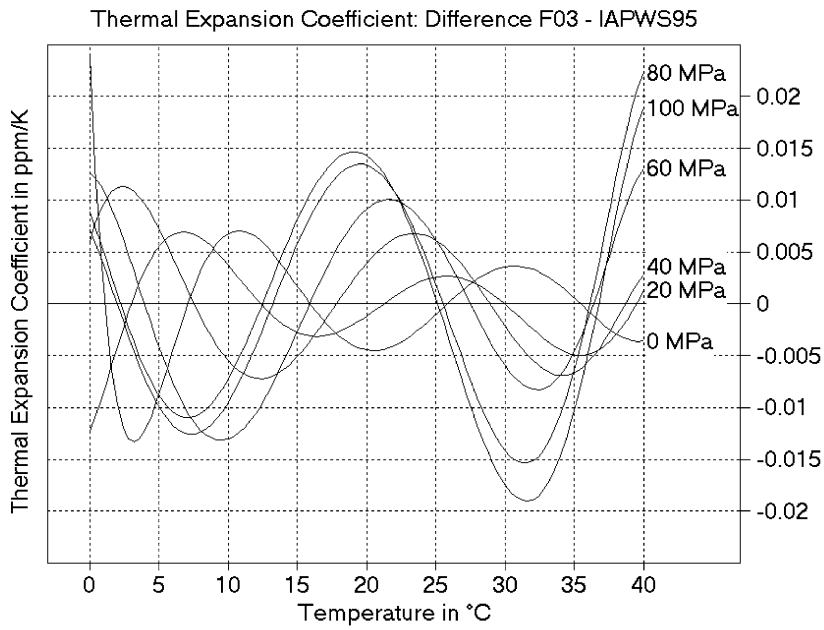

Fig. 4. Deviation of isobaric thermal expansion coefficients, $\alpha(0,t,p)$, of F03 from IAPWS95 at applied pressures between 0 and 100 MPa (0 and 10,000 dbar).

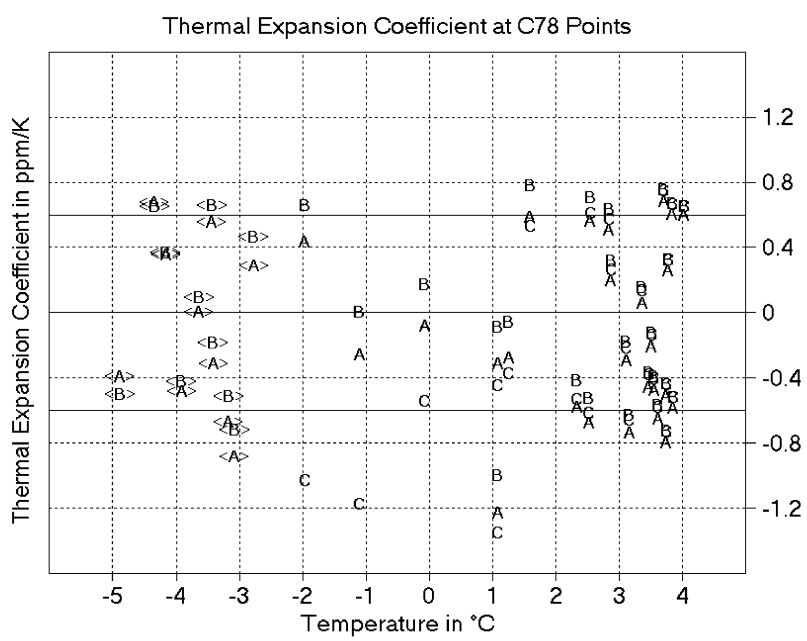

Fig. 5. Isobaric thermal expansion coefficients, $\alpha(0,t_{MD},p)$, at Caldwell's (1978) measured points of maximum density, t_{MD}, of water, computed with EOS80 (A), FH95 (B), and IAPWS95/F03 (C). Symbols in brackets are below the freezing temperature. The experimental error of Caldwell's measurements is 0.6 ppm/K, shown as solid lines.

Table 2
Comparison of temperatures (t_{MD}, ITS-90) and densities of pure water at its density maximum under atmospheric pressure taken from several sources or formulas

Source/Formula	t_{MD} /°C	ρ/(kg/m³)
F03	3.978 890	999.974 877
Tanaka et al., 2001	3.983 035	999.974 950
IAPWS95	3.978 121	999.974 873
FH95	3.978 102	999.974 657
Bradshaw and Schleicher, 1986	3.969 783	999.972 604
EOS80	3.980 759	999.974 961
Caldwell, 1978	4.02	–
Menaché, 1976	near to 4	999.975
Wagenbreth and Blanke, 1971	3.979 080	999.972 0

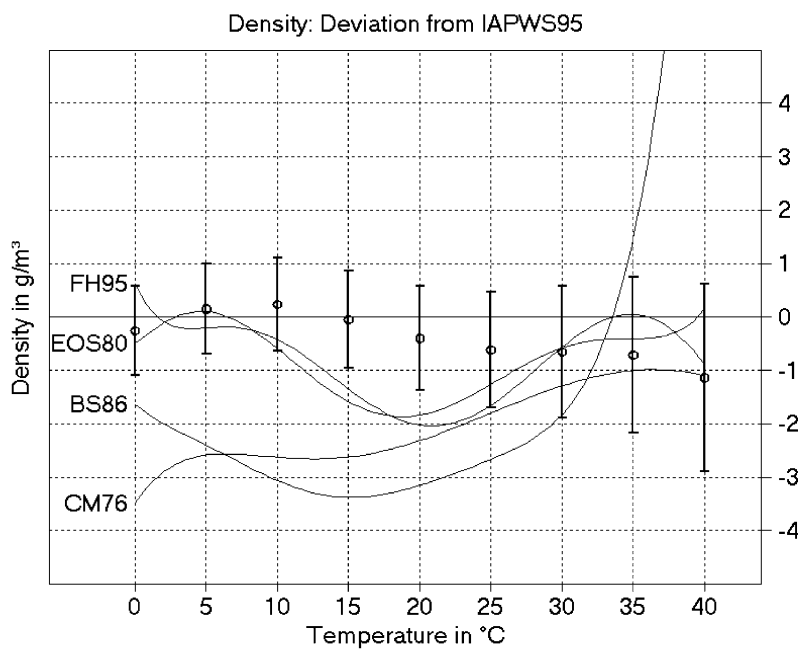

Fig. 6. Deviation of several water density formulas from IAPWS95 at atmospheric pressure (P = 0.101325 MPa). BS86 is the equation of state derived from specific volume measurements by Bradshaw and Schleicher (1986). CM76 describes related experiments of Chen and Millero (1976). Data points with error bars are values recently recommended by Tanaka et al. (2001).

EOS80 is within the error interval 30 ppm of IAPWS95, but IAPWS95 is clearly outside the range 4 ppm of EOS80 for pressures above 20 MPa.

The coefficients g_{00k}, $k > 0$ have been determined by fitting specific volume, Eq. (2), as,

$$\int\left\{\left(\frac{\partial g}{\partial p}\right)_{S=0,t} - \frac{1}{\rho^{IAPWS95}(T,P)}\right\}^2 dtdp = Min \tag{26}$$

with respect to reciprocal density given by IAPWS95, as given in appendix. The fit was jointly done with those for thermal expansion (section 3) and compressibility (section 5). The least-square integrals were

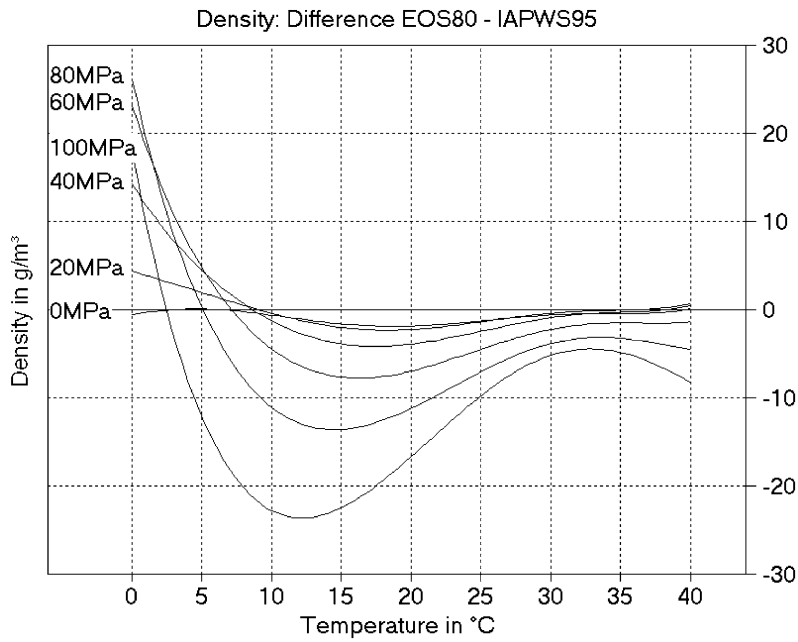

Fig. 7. Comparison of the high pressure equations of state EOS80 (claimed accuracy 4 ppm) and IAPWS95 (claimed accuracy 30 ppm) for applied pressures from 0–100 MPa (0–10,000 dbar).

extended between triple point and 110 MPa in applied pressure and from freezing points to 45 °C in temperature. Two data groups were used, one at $p = 0$ MPa with required variance of 1 ppm (resulting r.m.s. 0.021 ppm), and the other one for all pressures, requiring 30 ppm (resulting r.m.s. 0.032 ppm). The final maximum deviation between both functions is below 0.1 ppm, see Fig. 8, which is significantly less than the experimental uncertainty.

Maximum densities are preferably used for absolute calibration of density formulas. Table 2 is a list of densities computed or measured at their t_{MD}. The currently best value is $t_{MD} = 3.983\ 035 \pm 0.000\ 67$ °C with the density $\rho_{MD} = 999.974\ 950 \pm 0.000\ 84$ kg/m^3 (Tanaka et al., 2001, Wagner and Pruß, 2002). While their agreement in densities is excellent, all t_{MD} values are below that of Tanaka et al. and beyond its uncertainty. In the computation of F03, full compatibility with IAPWS95 was preferred over a better agreement with Tanaka's et al. data.

5. Compressibility of water

The new IAPWS95 formulation deviates from EOS80 and FH95 in isothermal compressibilities, Eq. (5), up to $\Delta K \approx 0.8$ ppm/MPa, which is twice the estimated experimental error (Kell and Whalley, 1975), see Fig. 9. Consequently, deviations in sound speeds about $\Delta U \approx 1$ m/s must be expected for high pressures and low temperatures, compare Eq. (29) and Fig. 11.

Compressibilities were adjusted to IAPWS95 by minimizing the least-square integral

$$\int\left\{\left(\frac{\partial^2 g}{\partial p^2}\right)_{S=0,t} + \frac{1}{(\rho^{IAPWS95})^2\cdot(\partial P^{IAPWS95}/\partial\rho)_T}\right\}^2 dtdp = Min \tag{27}$$

in a common fit together with thermal expansion coefficients (section 3) and densities (section 4). The

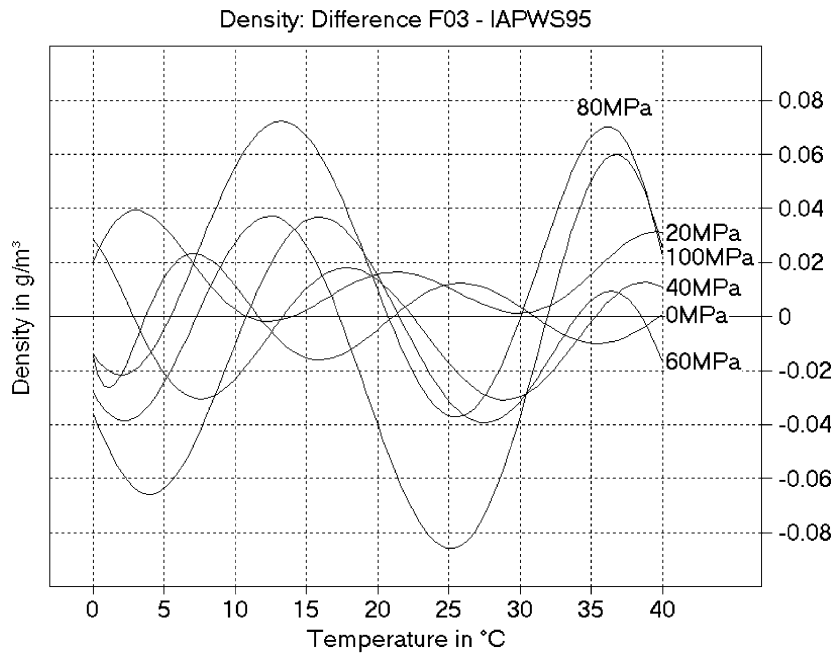

Fig. 8. Deviation between high pressure densities of F03 and IAPWS95 at applied pressures from 0–100 MPa (0–10,000 dbar).

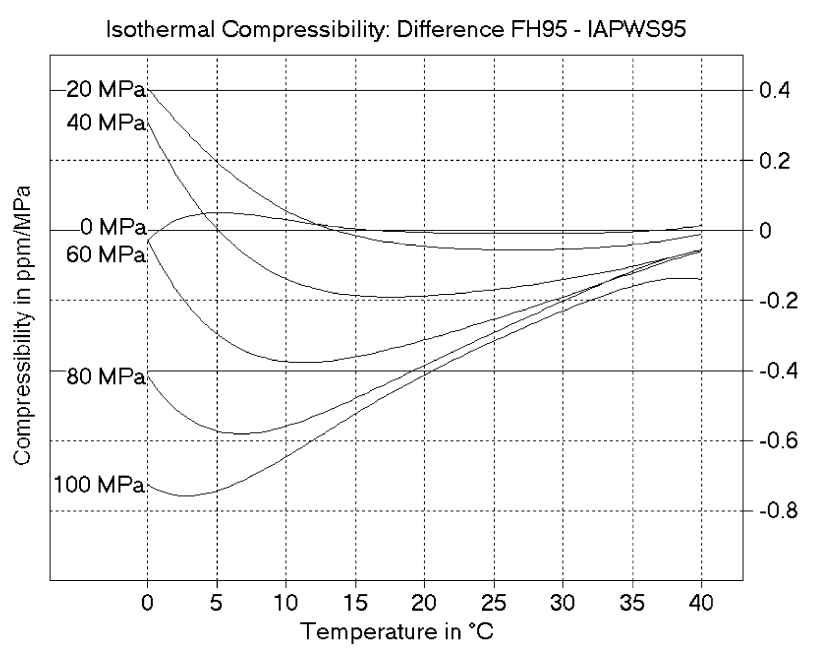

Fig. 9. Difference in isothermal compressibilities of water, $K(0,t,p)$, computed by FH95 and IAPWS95, for applied pressures p between 0–100 MPa (0–10,000 dbar). Estimated experimental accuracy is 0.4 ppm/MPa, shown by solid lines. Deviations between EOS80 and IAPWS95 are very similar.

integral was extended in pressure from the triple point to 110 MPa and from freezing temperatures up to 45 °C with required r.m.s. of 0.4 $mm^3/(kg\ MPa)$, resulting in a residual of 0.003 $mm^3/(kg\ MPa)$, Fig.10.

The regression runs done so far have determined all coefficients required for the so-called PVT properties of water. Other quantities like the adiabatic lapse rate or sound speed can now be directly computed by means of the corresponding thermodynamic relations.

For computed sound speed U, derived from adiabatic compressibility, Eq. (9),

$$U^{-2} = \left(\frac{\partial\rho}{\partial p}\right)_{S,\sigma} = K\rho + \frac{\alpha^2 T}{c_P} \tag{28}$$

the possible error can be estimated as,

$$\frac{\Delta U}{U} = 0.5\cdot U^2\left\{K\rho\left(\frac{\Delta v}{v}-\frac{\Delta K}{K}\right) + \frac{\alpha^2 T}{c_P}\left(\frac{\Delta c_P}{c_P}-\frac{2\Delta\alpha}{\alpha}\right)\right\} \tag{29}$$

or, with U = 1500 m/s, T = 300 K, K = 500 ppm/MPa, and the other values chosen as in Eq. (23),

$$\frac{\Delta U}{U}\approx 0.5\cdot\left(\frac{\Delta v}{v}-\frac{\Delta K}{K}\right) + 0.001\cdot\left(\frac{\Delta c_P}{c_P}-\frac{2\Delta\alpha}{\alpha}\right) \tag{30}$$

The biggest uncertainty of $\Delta U/U\approx$400 ppm or $\Delta U\approx$0.6 m/s is caused here by isothermal compressibility K with assumed accuracy $\Delta K\approx$0.4 ppm/MPa (Kell and Whalley, 1975), while the possible heat capacity error contributes only to negligible $\Delta U/U\approx$3 ppm.

Some error estimates applicable to EOS80 and FH95 sound speeds are 0.2 m/s (Wilson, 1959), 0.3 m/s (Barlow & Yazgan, 1967, Kell and Whalley, 1975), and for low pressures 0.03—0.04 m/s (Wille, 1986)

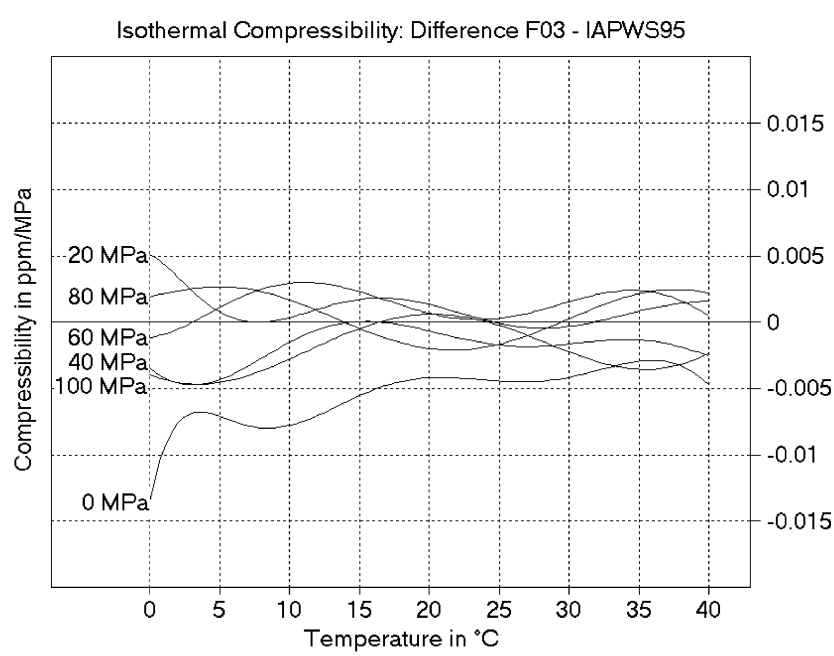

Fig. 10. Isothermal compressibility of water, $K(0,t,p)$, of F03 compared to IAPWS95 for applied pressures p between 0 and 100 MPa (0 and 10,000 dbar) as indicated at the curves. Experimental uncertainty is about 0.4 ppm/MPa.

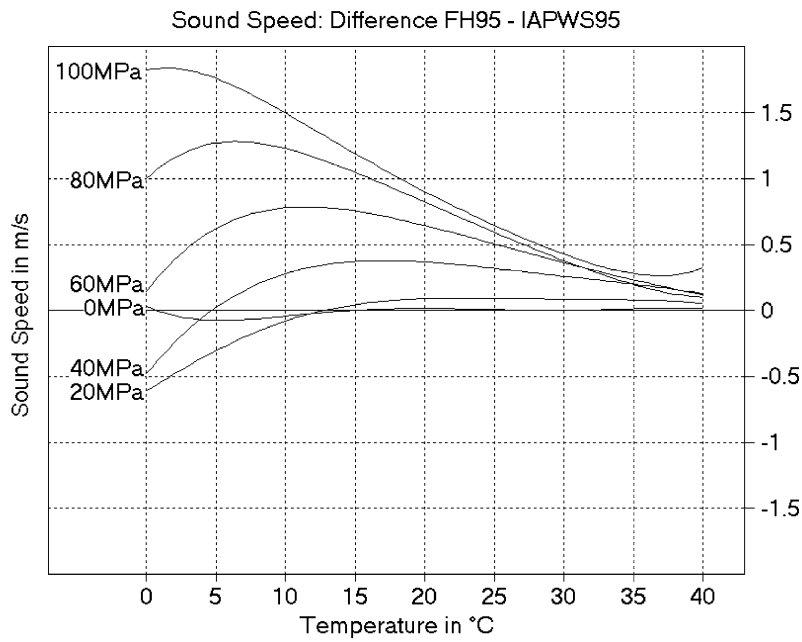

Fig. 11. Difference between the sound speeds of water, $U(0,t,p)$, computed by the FH95 and IAPWS95 formulas for applied pressures p between 0 and 100 MPa (0 and 10,000 dbar). Corresponding curves for the EOS80 or Kell and Whalley (1975) formulas look quite similar to FH95.

or 0.015 m/s (Del Grosso & Mader, 1972). They evidently underestimate the corrections required for EOS80 and FH95 at pressures above 40 MPa to meet the new IAPWS95 standard.

The sound speed equation of EOS80 is derived from measurements of Chen and Millero, (1977), who used as pure water reference the one atmosphere measurements of Del Grosso (1970) and Del Grosso and Mader (1972) together with the high pressure measurements of Wilson (1959). The latter ones are questionable (Del Grosso, 1974) and are mainly blamed (Millero and Li, 1994) to be the cause for discrepancies between computed and measured sound travel times in the ocean (Dushaw et al., 1993), which amount for about 0.5 m/s and are partly reduced but not overcome by the correction proposed by Millero and Li (1994), as was pointed out again by Meinen and Watts (1997). The sound speed formula of Del Grosso (1974), however, is in agreement with travel-time data within only 0.05 m/s. The new IAPWS95 sound speed formula suggested the hope that these problems with Chen-Millero sound speeds may now be eventually resolved in a natural way, but unfortunately this could not be achieved by a simple replacement of the pure water parts (see Fig. 12 and Table 3).

In the region of interest here, IAPWS95 sound speed can be considered as accurate as $\Delta U/U \approx 1000$ ppm (Wagner and Pruß, 2002) compared to measurements of Mamedov (1979) and Petitet, Tufeu & Le Neindre (1983). The precise measurements of Fujii (1994) between 303 and 323 K, and between 10 and 200 MPa, are even covered within $\Delta U/U \approx 250$ ppm (or 0.5 m/s). The one atmosphere measurements of Del Grosso and Mader (1972) and Fujii & Masui (1993) are reproduced with maximum errors of only 10 ppm (or 0.02 m/s) rsp. 30 ppm (or 0.05 m/s).

While the high-pressure data of Wilson (1959), which are suspected as erratic (Del Grosso, 1974, Millero and Li, 1994), are the only basis for the high pressure formulas of Kell and Whalley (1975), EOS80, or FH95, the new IAPWS95 standard is derived from more than 600 sound speed data points of 12 studies including only 32 from Wilson (Wagner and Pruß, 2002) and can therefore be considered as being much more reliable.

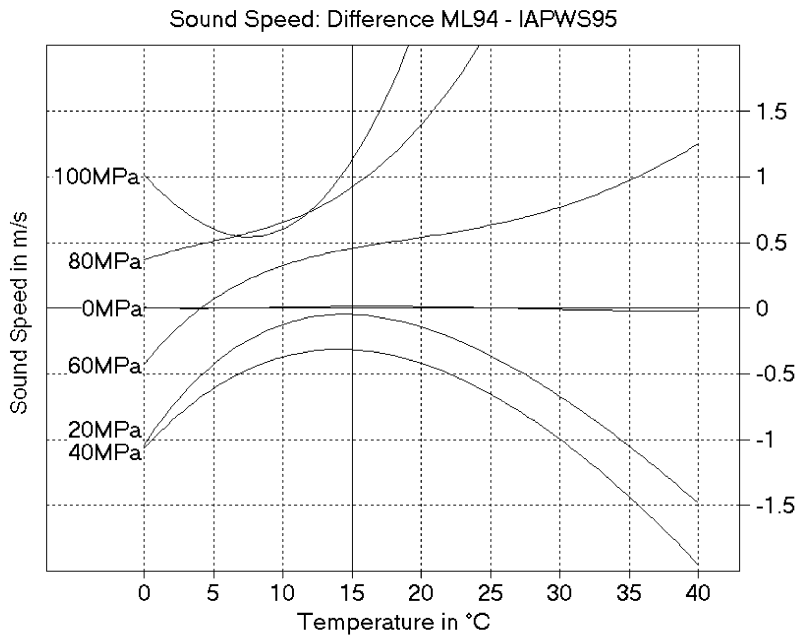

Fig. 12. Difference between the sound speeds of water, $U(0,t,p)$, computed by the ML94 equation (i.e. EOS80 with correction after Millero and Li, 1994) and IAPWS95 formulas, for applied pressures p between 0 and 100 MPa (0 and 10,000 dbar). ML94 is only applicable for temperatures t below 15 °C (solid line).

Table 3
Deviations of sound speed formulas from DG74 (Del Grosso, 1974) in the ranges of his Tables I to VII with different intervals of salinity, temperature and pressure. "ML94" is the correction of EOS80 sound speeds due to Millero and Li (1994). "Eq. (40)" means EOS80 sound speeds with its pure water part tentatively replaced by IAPWS95 sound speed (which is not valid for $t = 0$ °C at $p = 0$ MPa), as given in Eq. (40)

DG74 data	S	t_{68} °C	p MPa	EOS80 m/s	ML94 m/s	Eq. (40) m/s
Table I	29–41	0–35	0	0.086	0.086	0.101
Table II	33–37	0–35	2	0.205	0.102	0.236
Table III	29–43	0–30	0	0.066	0.066	0.074
Table IVa	29–43	0	5	0.221	0.094	0.483
Table IVb	29–43	10	2	0.068	0.030	0.110
Table IVc	29–43	20	1	0.026	0.036	0.030
Table IVd	29–43	30	0.1	0.138	0.129	0.164
Table V	33–37	0	0–100	0.601	0.070	0.813
Table VI	33–37	5	0–100	0.692	0.120	0.485
Table VII	0	0–30	0	0.036	0.036	0.039

Deviations between FH95 and IAPWS95 are small at atmospheric pressure but grow to almost 2 m/s for high pressures, as shown in Fig. 11. Very similar pictures (not shown here) arise for EOS80 or Kell-Whalley (1975) formulas if drawn instead of FH95 because all of them are derived from Wilson's (1959) high-pressure data.

The correction to Wilson (1959) as proposed by Millero and Li (1994) for temperatures up to 15 °C is shown in Fig. 12. Especially at pressures above 60 MPa the deviations are smaller by a factor of about 2 than those of FH95 (and EOS80), but still beyond the error of 0.5 m/s as discussed for seawater, above.

Sound speeds computed by Eq. (28) using the Gibbs potential derived in this paper deviate from IAPWS95 by typically 0.01 m/s or 10 ppm for all temperatures and pressures considered here, see Fig. 13, which is a well tolerable error.

6. Free enthalpy of water

Free enthalpy, $g(S,t,p)$, as proposed by Feistel and Hagen (1995) can immediately be compared with the corresponding expression derived from IAPWS95 free energy, $f(T,\rho)$, by $g = f+P/\rho$ after alignment of their reference states. While IAPWS95 entropy σ and internal energy e of the liquid water phase are supposed to vanish at the triple point $T_t = 273.16$ K, $P_t = 611.657$ Pa (Wagner and Pruß, 2002), entropy and enthalpy of FH95 are zero for standard seawater, $t_0 = 0$ °C ($T_0 = 273.15$ K), $p_0 = 0$ MPa ($P_0 = 0.101325$ MPa), compare Feistel and Hagen (1995). Both free enthalpies can differ by an arbitrary linear function of temperature,

$$g^{IAPWS95} = g^{FH95} + A + B{\cdot}T \tag{31}$$

The coefficients A and B can e.g. be determined by comparing entropies and enthalpies at a specified mutual reference point (T, P), which we have chosen at triple point temperature, $T = T_t$, and atmospheric pressure, $P = P_0$, to obtain the relative gauge constants $A = 61.034755$ J/kg and $B = 0.147566807$ J/(kg K).

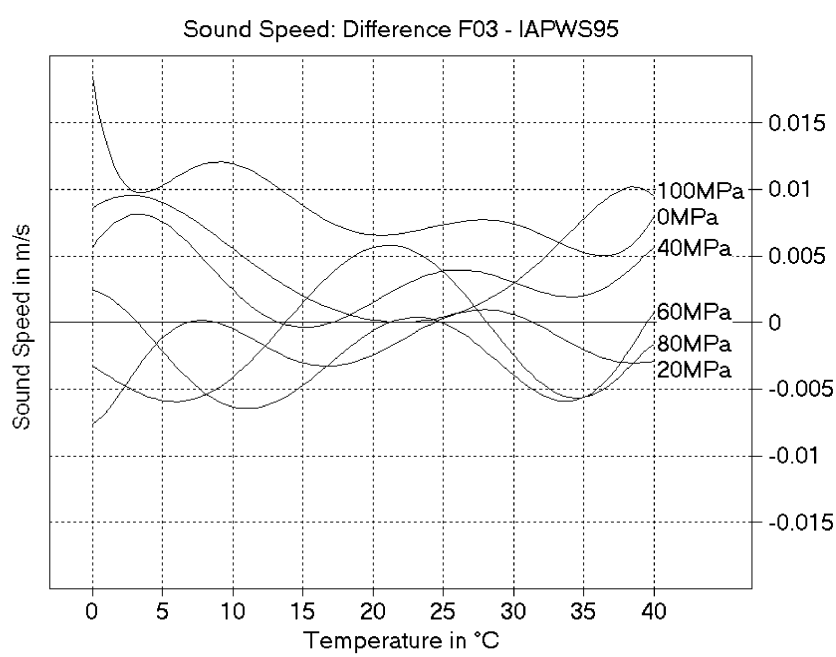

Fig. 13. Deviation between sound speeds of water, $U(0,t,p)$, computed by F03 and IAPWS95, for applied pressures p between 0 and 100 MPa (0 and 10,000 dbar).

The difference between both sides of Eq. (31) is shown in Fig. 14(a). While the curves belonging to different applied pressures between $p = 0$ and $p = 100$ MPa look very similar, both free enthalpies differ systematically and much stronger in their dependencies on temperature. This is caused by the offset in heat capacities as discussed in section 2.

Deviations in enthalpy $h = g - T\frac{\partial g}{\partial T}$ between the two sides of Eq. (32),

$$h^{IAPWS95} = h^{FH95} + A \tag{32}$$

see Fig. 14(b), following from the heat capacity offset as well, are in the order of $\Delta h \approx 100$ J/kg or $\Delta h/h \approx 0.1\%$, with only small consequences to be expected for e.g. oceanographic energy budgets.

Two coefficients g_{000}, g_{010} have been determined by adopting the pure water reference state of IAPWS95, which is vanishing entropy and internal energy at the triple point (t_t, p_t):

$$\left(\frac{\partial g(0,t_t,p_t)}{\partial t}\right)_{S,p} = 0 \tag{33}$$

$$g(0,t_t,p_t) - T_t\cdot\left(\frac{\partial g(0,t_t,p_t)}{\partial t}\right)_{S,p} - P_t\cdot\left(\frac{\partial g(0,t_t,p_t)}{\partial p}\right)_{S,t} = 0 \tag{34}$$

Although not mandatory, it is reasonable here to substitute the pure water reference state used in FH95 by a description consistent with the IAPWS95 standard. This change has become possible now because the F03 is valid at the triple point, while FH95 was not. The resulting maximum deviations over the whole range of pressures and temperatures are about 0.003 J/kg in free enthalpies, and 0.3 J/kg in enthalpies, see Fig. 15. These coefficients are required if computations are performed for typical cases like phase equilibria in freezing point or vapour pressure investigations.

7. Density of seawater

The EOS80 density equation $\rho^{EOS80}(S,t,p)$ of seawater at one atmosphere ($p = 0$ MPa) was determined by Millero and Poisson (1981a) with precision $\Delta\rho/\rho \approx 3.6$ ppm relative to pure water density, ρ_0, using the IPTS-68 temperature scale. They had used 122 data points of Millero, Gonzalez & Ward (1976a) and 345 data points from Poisson, Brunet & Brun-Cottan (1980). We have used here their normalized data $\Delta\rho = \rho - \rho_0$ as reported in Millero & Poisson (1981b), and minimized the corresponding expression

$$\sum\left\{\left(\frac{\partial g}{\partial p}\right)_{S,t} - \frac{1}{\Delta\rho(S,t_{68},0) + \rho^{IAPWS95}(T,P_0)}\right\}^2 = Min \tag{35}$$

with required r.m.s. deviation of 4 mm^3/kg to determine the coefficients g_{ij1}, $i > 0$, as shown in the appendix. At temperatures below the fresh water freezing point, IAPWS95 density is not valid and was extrapolated from $t = 0.0026$ °C by means of the thermal expansion coefficient. The latter correction is small (0.3 ppm). The temperatures t_{68} were converted into ITS-90 temperatures, t, by the formula given by Blanke (1989). The results of the fit are shown in Fig. 16. The regression polynomial fits well within the experimental scatter of 4 ppm. The r.m.s. deviation of the fits was 4 mm^3/kg for both the data sets of Poisson et al. (1980) and of Millero et al. (1976a). The relative densities $\Delta\rho = \rho(S,t,0) - \rho(0,t,0)$ of seawater to water computed by either EOS80 or F03 are shown in Fig. 17. Except some edge effects, both formulas agree fairly well within 4 ppm.

For higher salinities, the density measurements of Poisson & Gadhoumi (1993), see also Poisson et al. (1991), at temperatures 15–30 °C and salinities 34–50 have been included in a similar manner. EOS80

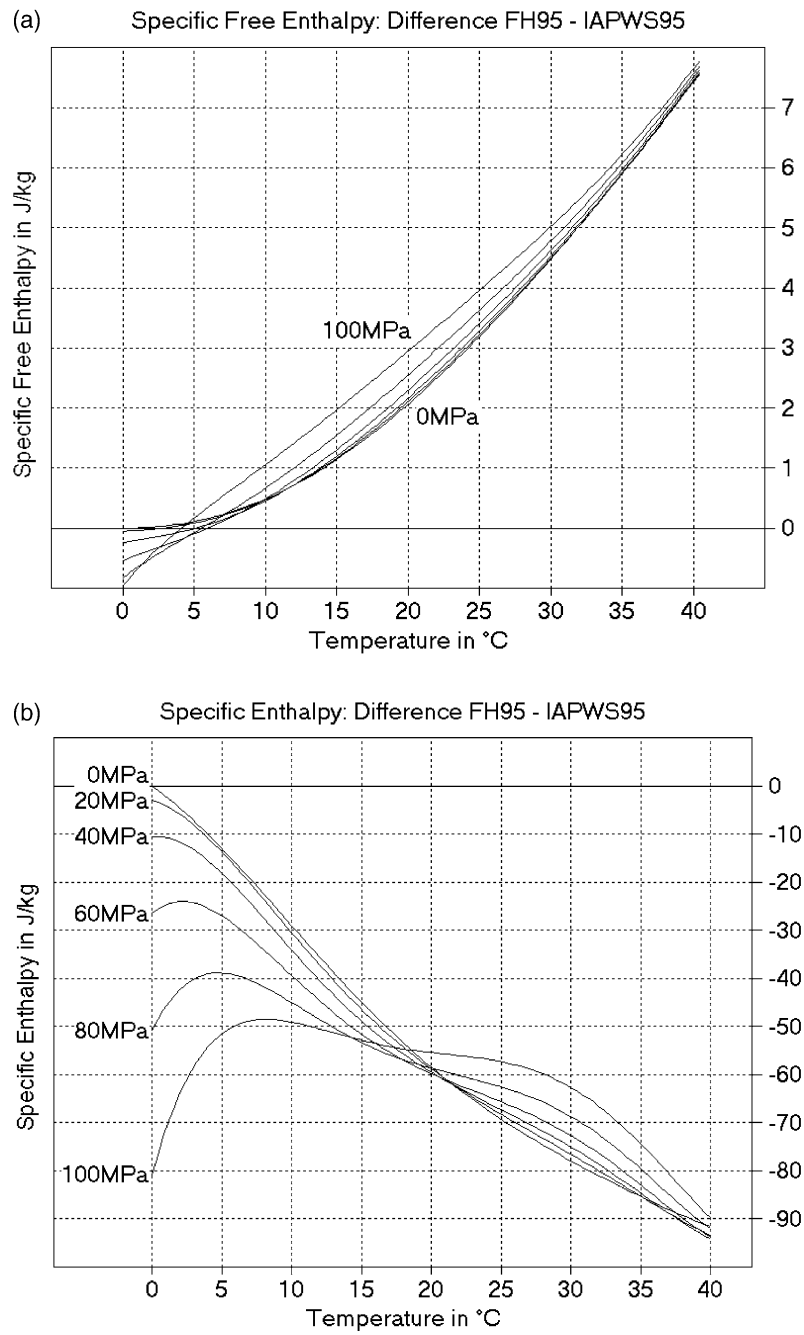

Fig. 14. Difference between pure water (a) free enthalpies, $g(0,t,p)$, above, and (b) enthalpies, $h(0,t,p)$, below, of FH95 and IAPWS95, after alignment to the same reference state, Eq. (31). The different curves belong to applied pressures p between 0 and 100 MPa. Neither enthalpy nor free enthalpy are available from EOS80. The striking differences in curvature of $g(0, t, p)$ and in slope of $h(0, t, p)$ are due to the different heat capacities involved.

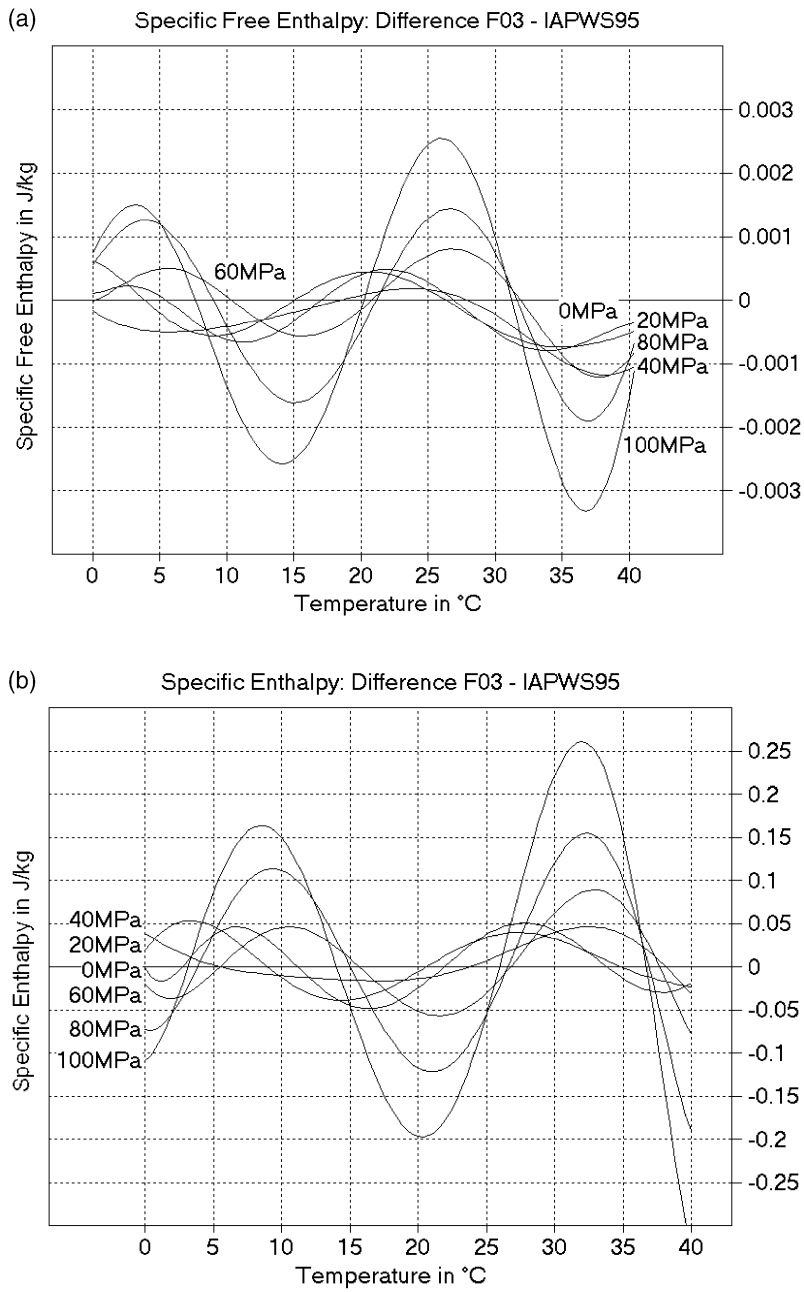

Fig. 15. Difference between (a) free enthalpies, $g(0, t, p)$, above, and (b) enthalpies, $h(0, t, p)$, below, of F03 and IAPWS95 for water at the applied pressures p as indicated at the curves. All functions refer to the same reference state, zero energy and zero entropy at the triple point.

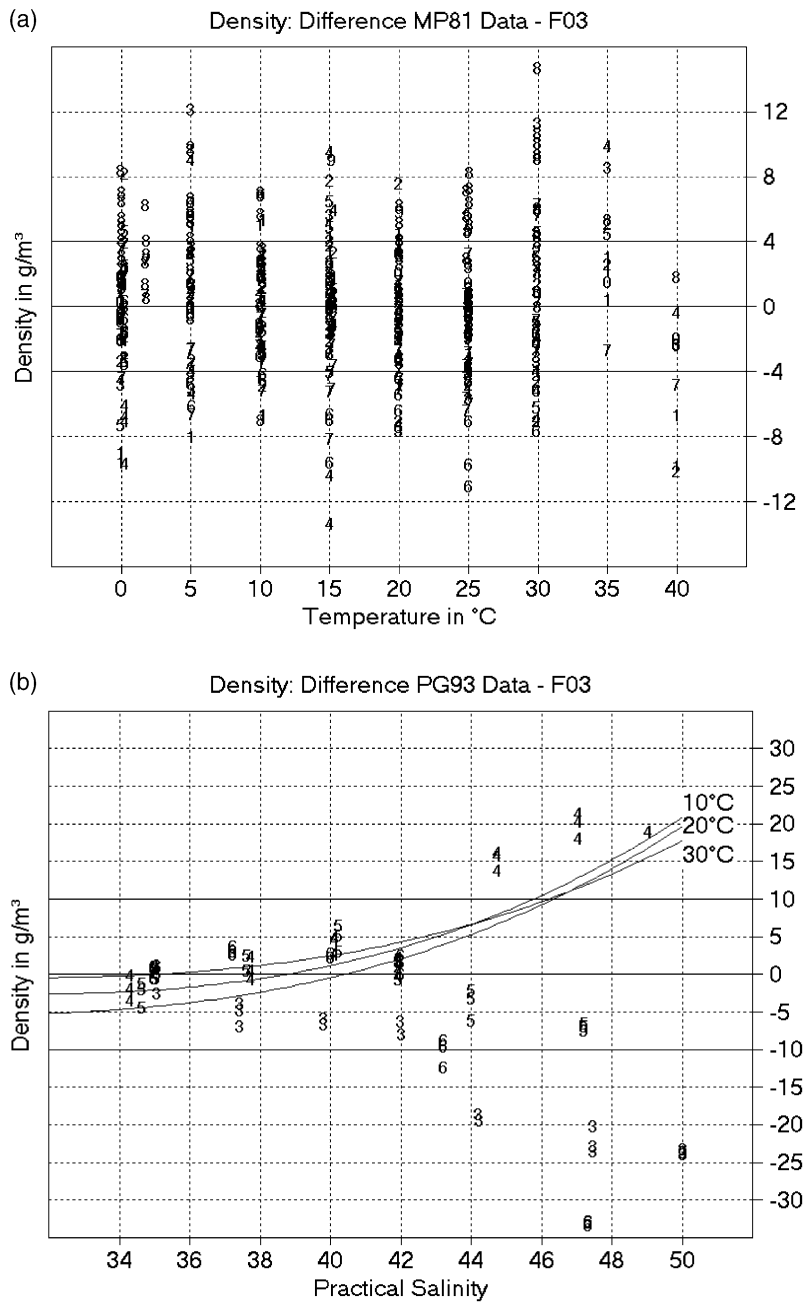

Fig. 16. Deviation of recalibrated density measurements at atmospheric pressure (P = 0.101325 MPa) from computed densities of F03: (a) MP81 data, above, as reported by Millero and Poisson (1981b). Symbols 0–8 represent salinities by S/5 rounded to integers. Experimental uncertainty of 4 ppm is indicated by horizontal lines. (b) PG93 data, below, as reported by Poisson and Gadhoumi (1993). Symbols 3–6 represent temperatures by t/5 rounded to integers. Data with $S > 44$ at 20 °C (“4”) appear about 30 ppm denser than the rest. Required tolerance 10 ppm of the fit is indicated by horizontal lines. Curves are computed by EOS80 for temperatures 10, 20 and 30 °C, called the ‘Red Sea extension’ up to S = 43 (Mamayev et al., 1991).

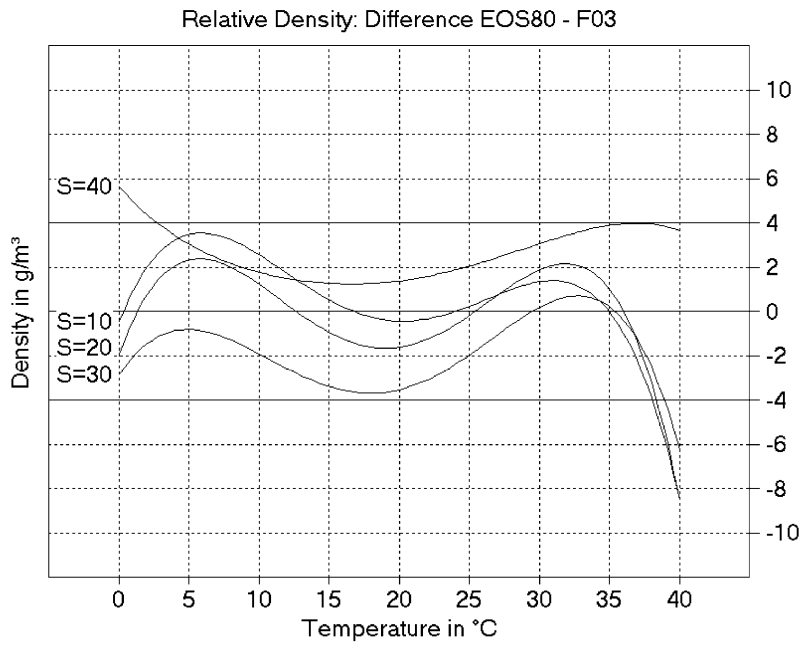

Fig. 17. Relative densities $\Delta\rho = \rho(S,t,0) - \rho(0,t,0)$ of seawater to pure water have been computed by EOS80 and by F03 for various salinities S (as indicated at the curves) as functions of temperature t.

pure water densities have first been subtracted from densities reported in the paper, and the resulting relative densities were treated as in Eq. (35). The error range of 4 ppm claimed by the authors had to be relaxed to merely required 10 ppm due to apparent conflicts with other data groups in this fit, which finally achieved an r.m.s. of 11 ppm. The data at 20 °C and salinities above 44 seem to be responsible for this discrepancy since they form a separate cluster in Fig. 16(b).

Because the same coefficients also influence heat capacities, density maxima, sound speeds, and high-pressure densities, a simultaneous regression of all these quantities was performed. The other fits are discussed in the following sections.

8. Heat capacity of seawater

The EOS80 heat capacity equation is derived from measurements of Millero et al. (1973a). They determined the power ratios $r(S,t) = \Delta P/P$ of their Picker calorimeter with an overall uncertainty of 0.5% and computed heat capacities $c_P(S,t,0)$ of seawater at atmospheric pressure by the formula,

$$\frac{c_P(S,t,0)}{c_P(0,t,0)} = (1 + r(S,t)) \cdot \frac{\rho(0,t,0)}{\rho(S,t,0)} \tag{36}$$

using densities $\rho(S,t,0)$ and $\rho(0,t,0)$ of EOS80 and pure water heat capacities $c_P(0,t,0)$ as reported by Stimson (1955). Fortunately, in their paper they have made available their measured power ratios such that a recalibration of their heat capacities could easily be done. We have minimized the corresponding sum over all reported points $r(S,t_{68})$, as,

$$\sum \left\{ T\left(\frac{\partial^2 g}{\partial t^2}\right)_{S,p=0} + c_P^{IAPWS95}(T,P_0)\cdot(1 + r(S,t_{68}))\cdot\rho^{IAPWS95}(T,P_0)\cdot v(S,t,0) \right\}^2 = Min \tag{37}$$

with a required r.m.s. deviation of 0.5 J/(kg K) (Millero et al., 1973a) using specific volume $v = \left(\frac{\partial g}{\partial p}\right)_{S,t}$ available after Eq. (35) to determine the coefficients g_{ij0}, $i > 0, j > 1$. The fit was carried out simultaneously with further heat capacity data, below, as well as the ones for density, sound speed and density maxima due to commonly required coefficients of the thermodynamic potential. The results are shown in Fig. 18. The current fit reproduces the recalibrated measurements with an r.m.s. of 0.5 J/(kg K). Former equations EOS80 and FH95 deviate from it up to 5 J/(kg K), or 0.1%.

Bromley et al. (1970) have determined heat capacities at atmospheric pressure for seawater over a wide range of temperatures and salinities. These data can be assumed to be very accurate, compare Fig. 6. The authors estimated the maximum error to be 0.22% or 9 J/(kg K). Systematic thermal causes may possibly contribute one half to it. We have selected 25 points from t = 0–40 °C and from S = 10–50. The latter ones were used especially to support the high salinity density data of Poisson and Gadhoumi (1993), see section 7. Assuming that systematic thermal errors of the calorimeter bomb are likely independent of sample salinities, they may be diminished by using as reference the heat capacities of pure water reported by Bromley et al. (1970) in comparison to IAPWS95. Therefore, we have adjusted their seawater data, $c_p^{BDSW70}(S,t_{48},0)$, supposing IPTS-48 temperature scale, prior to regression as,

$$\sum\left\{T\cdot\left(\frac{\partial^2 g}{\partial t^2}\right)_{S,p=0} + c_P^{BDSW70}(S,t_{48},0)\cdot\frac{c_P^{IAPWS95}(T,P_0)}{c_P^{BDSW70}(0,t_{48},0)}\right\}^2 = Min \tag{38}$$

These recalibration corrections remain between 0.02 and 0.08% of the measured values, thus, the function $c_p^{BDSW70}(0,t_{48},0)$, based on data of Osborne et al. (1939), agrees quite well with IAPWS95 heat capacities.

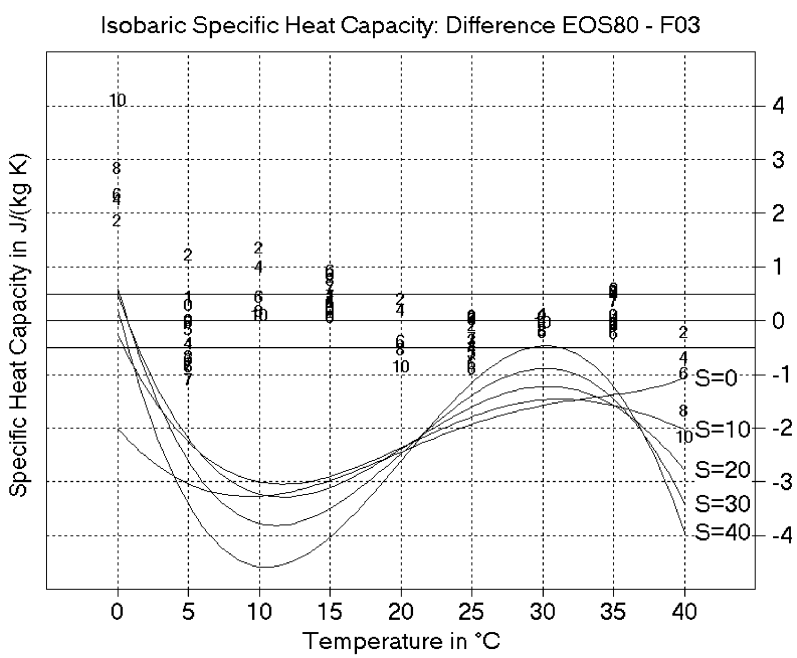

Fig. 18. Comparison of EOS80 isobaric specific heat capacities at atmospheric pressure (P = 0.101325 MPa), $c_P(S,t,0)$, with F03 for salinities S = 0–50 as indicated at the curves. Symbols 1–10 indicate, by salinities as S/5 rounded to integers, the recalibrated measurements at atmospheric pressure of Millero et al. (1973a) at temperatures t = 5, 15, 25, 35 °C and of Bromley et al. (1970) at t = 0, 10, 20, 30 and 40°C. Estimated precision of relative heat capacity measurements is marked by solid lines. FH95 heat capacities, not shown, are very similar to the EOS80 curves.

For the fit, we have required an r.m.s. deviation of 2 J/(kg K) (or 0.05%), which was met by an even less resulting mean residual of 1.4 J/(kg K) (or 0.01%), compare Fig. 18. The data seem to possess a systematic, almost linear temperature trend in the plot, which contributes most of the scatter at the interval edges of 0 and 40 °C.

For higher pressures, computed heat capacities of EOS80 and FH95 deviate are up to 3 times as much, see Fig. 19, although still obeying the typical accuracy of 10 J/(kg K) of specific heat measurements. These stronger differences are mainly a result of a modified determination of high pressure densities in comparison to FH95, see section 11 for details.

9. Thermal expansion of seawater

Temperatures of maximum density, t_{MD}, determined by Caldwell (1978) for seawater of various pressures and salinities, correspond to zeroes of the thermal expansion coefficient. The experimental temperature uncertainty of 40 mK leads to a limitation of thermal expansion in between $\pm$ 0.6 ppm/K (Feistel and Hagen, 1995). The fit was done to minimize the sum over all reported triples (S, t_{MD}, p), assuming IPTS-68,

$$\sum_{t_{MD}} \left\{ \frac{\partial^2 g}{\partial t \partial p} \right\}^2 = Min \tag{39}$$

with a required r.m.s. deviation of 0.6 mm^3/(kg K), jointly with density, heat capacity and sound speed of seawater. The fit achieved the desired goal with a resulting r.m.s. of 0.7 mm^3/(kg K), compare Fig. 20. The current F03 formula reproduces the t_{MD} data as good as FH95, but visibly better than EOS80.

For higher temperatures and salinity S = 35, EOS80 thermal expansion coefficients deviate from the current ones up to 10 times the estimated experimental error, see Fig. 21. FH95 deviations are of similar magnitude, but happen already at lower temperatures. The main reason for this difference is the fact that FH95 high pressure densities outside the naturally encountered combinations of S, t and p are exclusively determined from EOS80 sound speeds, while EOS80 high-pressure densities are mainly based on measured thermal expansion data. On the other hand, for temperatures up to 20 °C and pressures up to 40 MPa (4000 dbar), all three formulas agree well within the tolerance range.

10. Sound speed in seawater

Sound speed measurements of seawater are used here to determine its compressibilities and high pressure densities. The function of Del Grosso (1974), briefly DG74, can immediately be used for this purpose after converting temperatures from IPTS-68 to ITS-90 scales. It is only valid for Neptunian waters, in form of his “tables” I–VI with accuracy 0.05 m/s, see Table 3. Due to this small error, these data are the preferred means for the determination of high-pressure densities since the precision of the results can be expected to be one order of magnitude more accurate than the direct density measurements (see Eq. (29)). Measurements of Chen and Millero (1977), briefly CM77, cover the entire range of salinities S = 0–40, temperatures 0–40 °C and pressures 0–100 MPa, but they were found to be in error up to 0.5 m/s (Dushaw et al., 1993, Millero and Li, 1994, Meinen and Watts, 1997). Millero and Li (1994), briefly ML94, pointed out that Chen and Millero had measured only sound speed differences to pure water, for which they had used the formula of Wilson (1959), and proposed a pressure correction valid below 15 °C.

We have studied a possible (simply additive) replacement of pure water sound speed in the EOS80 (CM77) formula by IAPWS95 sound speeds. The comparison made within the different tables of DG74 is shown in Table 3. We see that in the oceanographically important high-pressure regions of tables V and

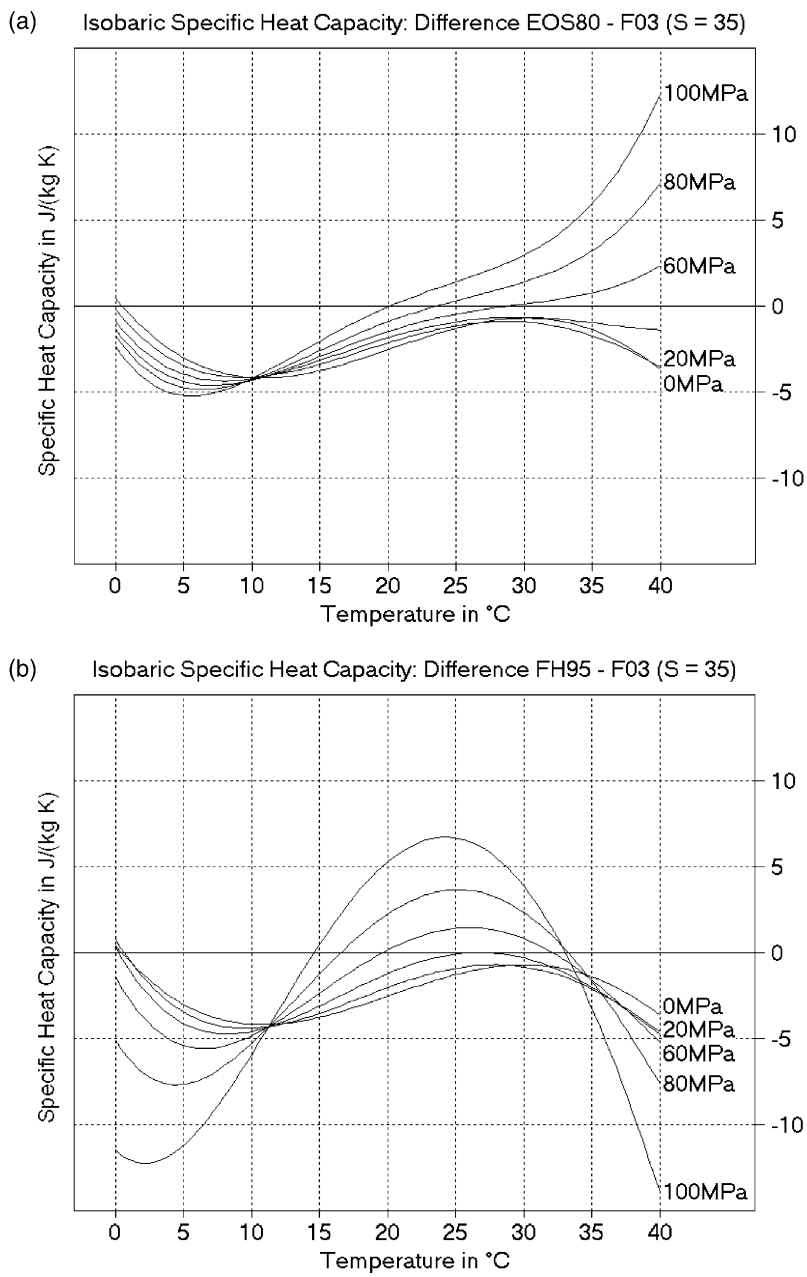

Fig. 19. High pressure isobaric specific heat capacities, $c_P(S,t,p)$, computed with (a) EOS80, above, and (b) FH95, below, compared with F03, for salinity $S = 35$ and applied pressures p between 0 and 100 MPa (0 and 10,000 dbar).

VI the mean deviation between DG74 and EOS80 exceeds 0.6 m/s, which is reduced by the ML94 correction to about 0.1 m/s. A tentative replacement of the pure water part, as suggested by the discussion of Millero and Li (1994), of EOS80 by IAPWS95 due to the formula,

$$U^{tent}(S,t,p) = U^{EOS80}(S,t_{68},p) - U^{EOS80}(0,t_{68},p) + U^{IAPWS95}(T,P) \quad (40)$$

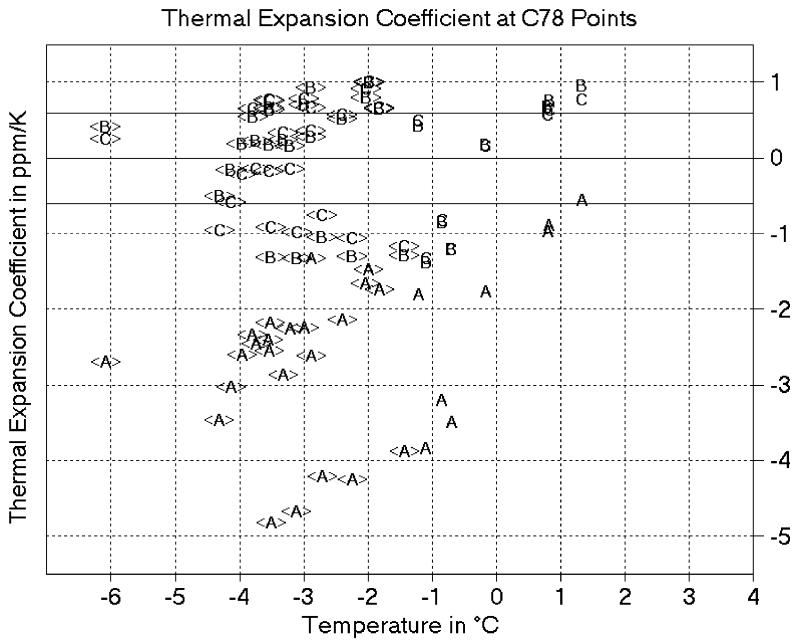

Fig. 20. Isobaric thermal expansion coefficients of seawater, $\alpha(S,t_{MD},p)$, at Caldwell's (1978) temperatures of maximum density, t_{MD}, for various pressures p and salinities S, computed with EOS80 (A), FH95 (B) and F03 (C). Experimental error bounds are indicated by solid lines. Symbols in brackets are below the freezing temperature.

does not improve the data quality relative to DG74, see Table 3.

For the regression determining the coefficients $g_{i,j,k}$, $i > 0$, we have used DG74 in the regions in S-t-p space of his tables (Table 3) to solve the nonlinear problem,

$$\int dS\,dt\,dp\left\{\frac{(\partial g/\partial p)^2\cdot\partial^2 g/\partial t^2}{(\partial^2 g/\partial t\partial p)^2-\partial^2 g/\partial t^2\cdot\partial^2 g/\partial p^2}-U^2(S,t_{68},p)\right\}^2 = Min \tag{41}$$

with required r.m.s. of 0.05 m/s.

In distinction to the previous FH95 Gibbs function computation, we have completely refrained this time from using CM77 sound speeds because of the virtual impossibility to recalibrate them properly to the IAPWS95 standard as shown in Table 3. The correction formula ML94 is neither covering the entire S-t-p region, nor is it sufficiently precise (Meinen and Watts, 1997). Instead, sound speeds of the regions in the S-t-p space outside of Del Grosso's Neptunian waters were indirectly determined here from high-pressure density data. These data proved as very well compatible with DG74 in their overlapping S-t-p domains, as shown in Fig. 22, and in section 11. The fit was carried out in one single run together with densities, heat capacities, and maximum density temperatures. As results, we obtained r.m.s. values for tables I–III of 0.017 m/s, for table IV of 0.012 m/s, and for tables V–VI of 0.035 m/s.

Fig. 22 shows the comparison of the current F03 fit with DG74, EOS80/CM77, SM91 (Spiesberger and Metzger, 1991), ML94 and FH95 formulas for the oceanographically most interesting tables V and VI of Del Grosso. SM91 is a pressure correction to DG74, presumably valid up to 40 MPa, derived from acoustic pulse travel times in the deep Pacific. Deviations from DG74 and FH95 remain in the negligible range of 0.05 m/s, while ML94 provides systematically higher values which exceed even 0.1 m/s. However, for the less interesting conditions of high pressures and high temperatures, not shown here, the situation is still unsatisfactory: deviations between different formulas exceed several m/s in extreme cases. The only system-

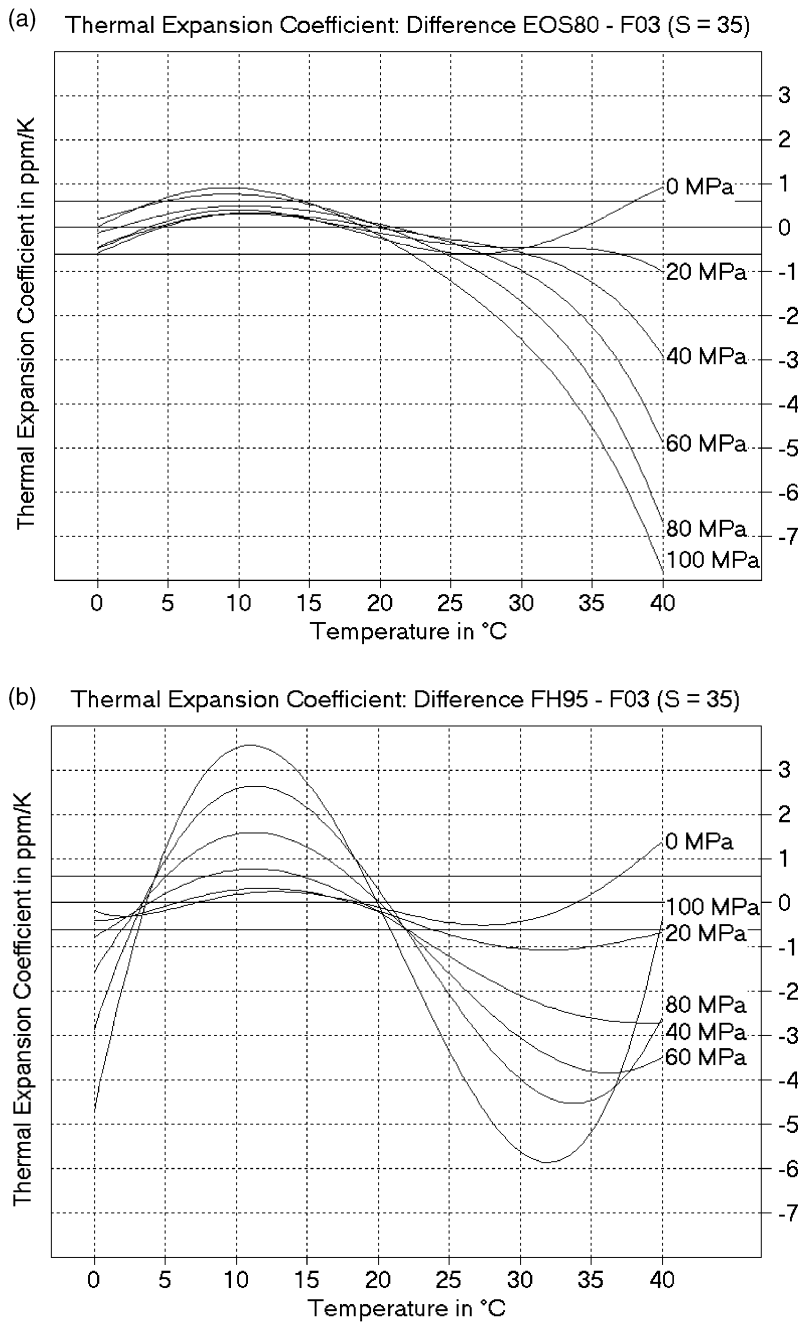

Fig. 21. Isobaric thermal expansion coefficients, $\alpha(S,t,p)$, of (a) EOS80, above, and (b) FH95, below, compared to F03, for seawater with salinity $S = 35$ and various pressures p, as indicated at the curves. Experimental uncertainty of 0.6 ppm/K is marked by horizontal lines.

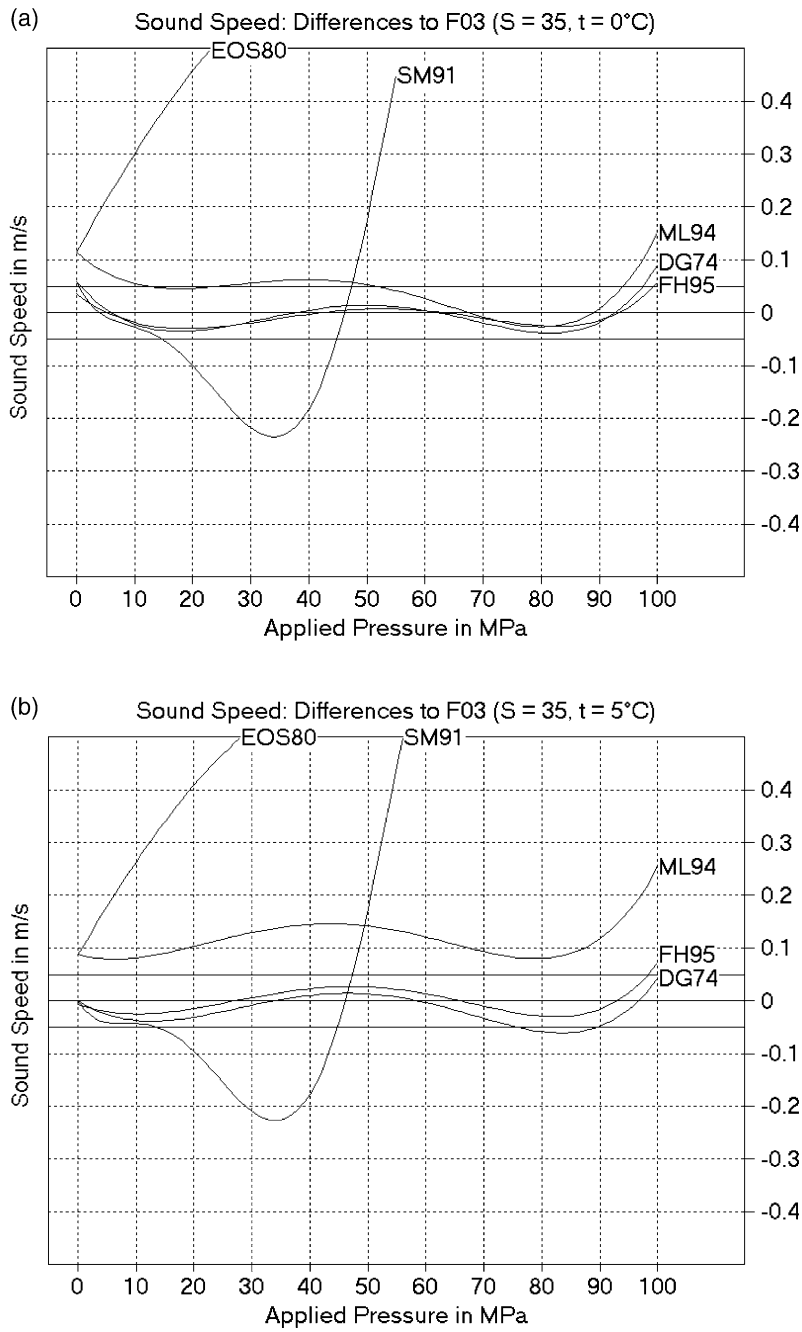

Fig. 22. Comparison of high pressure sound speeds, $U(S,t,p)$, of DG74 (Del Grosso, 1974), EOS80 (Chen and Millero, 1977), SM91 (Spiesberger and Metzger, 1991), ML94 (Millero and Li, 1994) and FH95 (Feistel and Hagen, 1995), with F03, for (a) table V, S = 35, t = 0 °C, above, and (b) table VI, S = 35, t = 5 °C, below, of Del Grosso. Estimated accuracy of 5 cm/s is indicated by solid lines.

atic measurements here are still the data of Chen and Millero (1977) which apparently lack of a proper calibration to high-pressure sound speeds of pure water (Millero and Li, 1994).

11. High-pressure density of seawater

The specific volumes of seawater at high pressures have been measured by Chen & Millero (1976) and were used for the construction of the EOS80 pressure dependence (Millero et al., 1980). They reported 558 readings from 0–40 °C, 0–100 MPa and S = 5–40 together with a regression polynomial $v^{CM76}(S,t_{68},p)$, which reproduces their data within an error range of 8.6 ppm, as shown in Fig. 23(b). The authors explicitly describe how the data are gauged to specific volumes of pure water, such that a new calibration with respect to the actual IAPWS95 water properties is relatively obvious. Requiring a mean deviation of 10 ppm, we have minimized the sum over all their data samples v^{meas} in the form of,

$$\sum\left\{\left(\frac{\partial g}{\partial p}\right)_{S,t}-\frac{v^{meas}(S,t_{68},p)}{v^{CM76}(0,t_{68},p)\cdot\rho^{IAPWS95}(T,P)}\right\}^2 = Min \tag{42}$$

The idea behind this correction is the assumption that the most significant errors come from the temperature-pressure calibration of the measuring device, and that these errors are independent of the salinity of the sample under study. Considering IAPWS95 pure water densities as “truth”, Eq. (42) removes all errors of this kind from the measured data since they appear equally in v^{meas} and v^{CM76}. The regression returned an r.m.s. of 11 ppm, as shown in Fig. 23(a), which is nearly as small as the experimental scatter itself. Thus, F03 describes these recalibrated measurements almost equivalently well as the regression polynomial of the authors, and at the same time consistently with Del Grosso’s (1974) sound speeds. Note that the r.m.s. deviation of the v^{meas} data from EOS80 is even bigger (17 ppm, Fig. 23(c)). EOS80 is considered as accurate as 9 ppm (Millero et al., 1980).

A second extended data set on high-pressure densities used for EOS80 was the thermal expansion experiment by Bradshaw and Schleicher (1970). They reported 221 specific volume data $\Delta v^{meas}(S,t,p) = v(S,t,p)-v(S,0,p)$ relative to 0 °C from −2–30°C (IPTS-48 scale), 1–100 MPa and S = 30–40. Their equipment was calibrated with mercury. With a required precision of 4 ppm as in the case of one-atmosphere densities (section 7) we have minimized the expression,

$$\sum\left\{\left(\frac{\partial g}{\partial p}\right)_{S,t}-\left(\frac{\partial g}{\partial p}\right)_{S,t=0}-\Delta v^{meas}(S,t_{48},p)\right\}^2 = Min \tag{43}$$

resulting in an r.m.s. deviation of only 2.6 ppm of the fit, as shown in Fig. 24. This scatter is comparable to that relative to the thermal expansion polynomial of those authors (3.2 ppm). The r.m.s. deviation of 6 ppm of these data from EOS80 is twice as big. We may conclude that by the current formulation F03, these data of Bradshaw and Scheicher(1970) could be brought into good agreement with recalibrated specific volumes of Chen and Millero (1976), with Del Grosso’s (1974) sound speeds, and with Caldwell’s (1978) t_{MD} data.

In the paper of Bradshaw & Schleicher (1976) the changes of volumes $V_i(S,t,P)$ due to pressure P of given seawater samples (i) with unknown amounts are reported. These data were not included into the fit of F03. For comparison of these measurements with the current formulas, we have first determined the sample masses M_i by regression with respect to the actual specific volume polynomial, $v = \partial g/\partial p$, as,

$$\sum\left\{\left(\frac{\partial g}{\partial p}\right)_{S,t}\cdot M_i-\frac{V_i(S,t_{48},P)}{v^{BS86}(t_{68},p)}\cdot\left(\frac{\partial g}{\partial p}\right)_{S=0,t}\right\}^2 = Min \tag{44}$$

The resulting specimen masses are shown in Table 4.

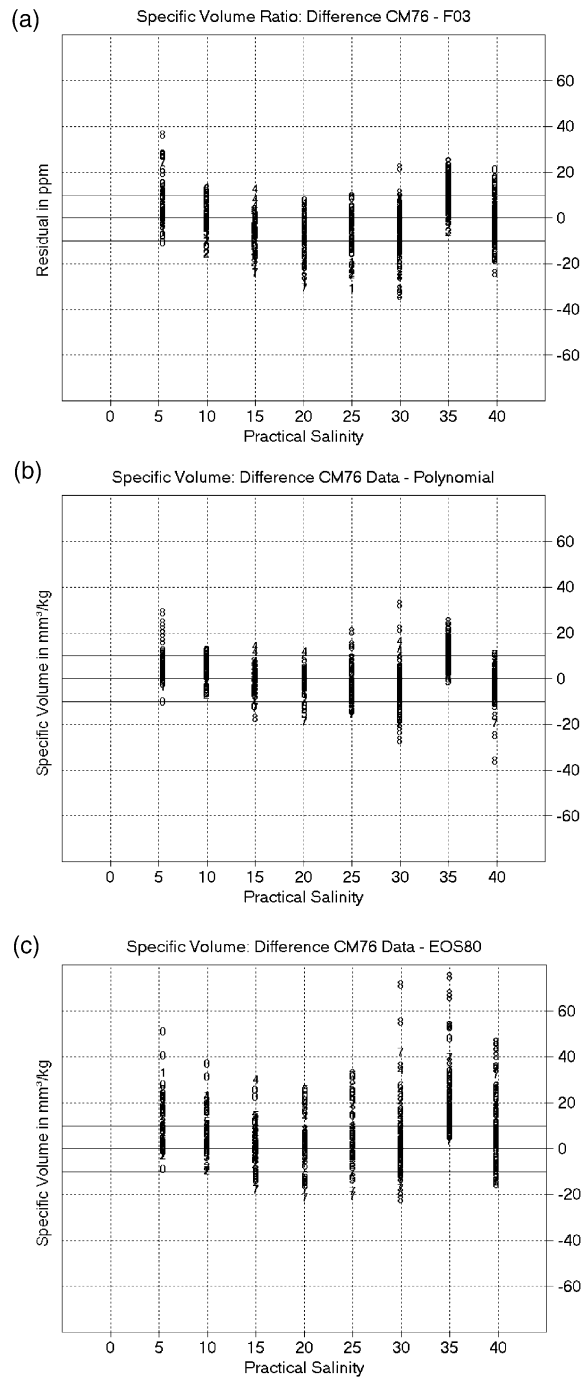

Fig. 23. Specific volume data of Chen and Millero (1976), $v^{meas}(S,t_{68},p)$, (a) above, compared with F03 after recalibration to pure water, in the way used in Eq. (42), $\frac{v^{meas}(S,t_{68},p)}{v^{CM76}(0,t_{68},p)}-\frac{v(S,t,p)}{v(0,t,p)}$, (b) middle, compared with the regression polynomial, $v^{CM76}(S,t_{68},p)$, of Chen and Millero (1976), (c) below, compared with EOS80. Symbols 0–8 used reflect temperatures by $t/5$ rounded to integers. Horizontal lines at 10 ppm indicate the r.m.s. required for the current fit.

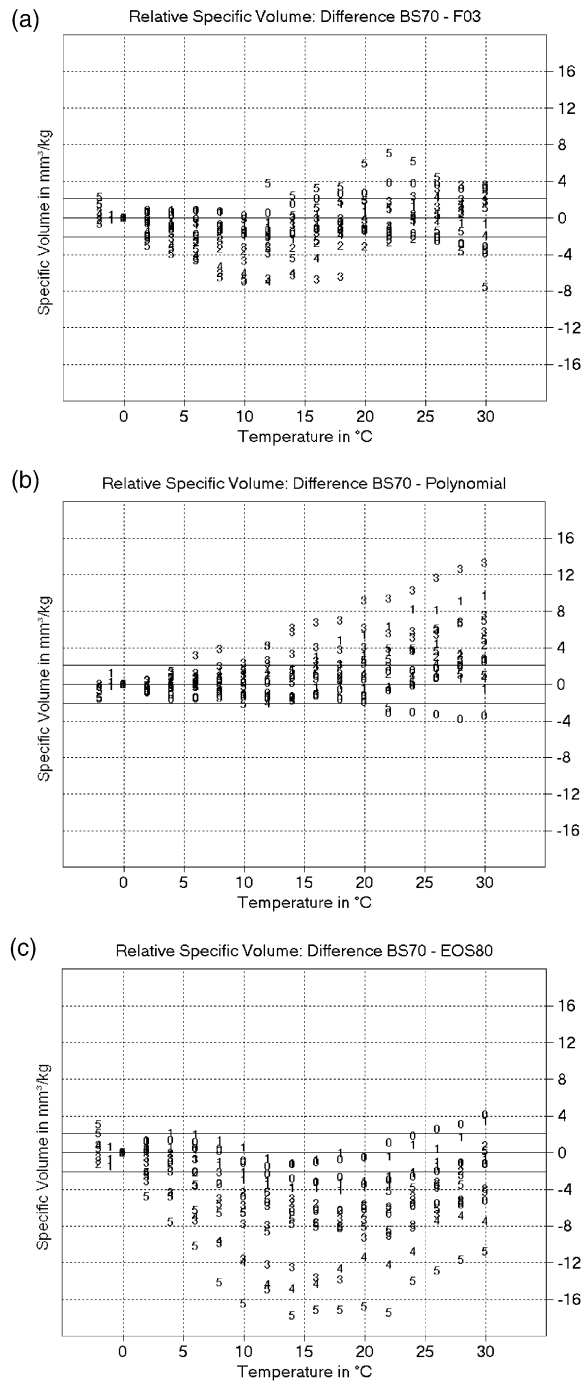

Fig. 24. Comparison of relative specific volume measurements of Bradshaw and Schleicher (1970), $\Delta v^{meas}(S,t,p) = v(S,t,p) - v(S,0,p)$, with (a) F03, above, (b) the polynomial of Bradshaw and Schleicher (1970), middle, and (c) EOS80, below. The r.m.s. error range of 2 ppm required for the current fit is indicated by horizontal lines. Symbols represent the pressure as p/20MPa rounded to integers.

Table 4
Seawater samples 1–7 of Bradshaw and Schleicher (1976) used for compression measurements with their masses determined by regression

BS76 # i	S	t_{48} /°C	M_i /g
sample #1	0	10	43.2085
sample #2	30.705	10	47.7841
sample #3	34.891	10	48.2614
sample #4	38.884	10	49.0071
sample #5	34.897	10	48.3196
sample #6	34.897	25	48.3203
sample #7	34.912	25	48.3041

A good agreement with r.m.s. 4.8 ppm as shown in Fig. 25 was achieved after readjustment of the pressure dependence of pure water, in analogy to Eqs. (42) and (44), as,

$$v^{recalib}(S,t,p) = \frac{V_i(S,t_{48},P)/M_i}{v^{BS86}(t_{68},p)\cdot\rho^{IAPWS95}(T,P)} \tag{45}$$

Here, $v^{BS86}(t_{68},p)$ is the specific volume of pure water proposed by Bradshaw & Schleicher (1986).

The comparison of seawater densities with $S = 35$ from EOS80 and from FH95 with densities from this paper over the entire intervals of temperatures and pressures shows maximum deviations of 40 ppm, see

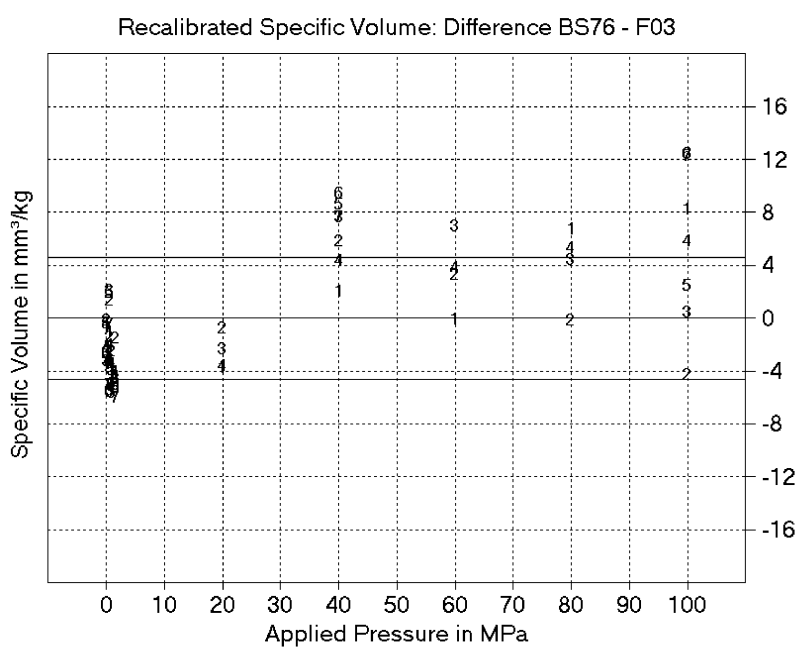

Fig. 25. Specific volume of compressed seawater samples reported by Bradshaw and Schleicher (1976), recalibrated after Eqs. (44), (45), $v^{recalib}(S,t,p)$, in comparison to F03. Symbols are sample numbers as given in Table 4. The r.m.s. deviation of 4.8 ppm is indicated by horizontal lines.

Fig. 26. Biggest errors appear in FH95 for high temperatures and pressures, while FH95 curves between 0 and 7 °C agree within 7 ppm with the current formula for all pressures. Deviations of EOS80 are more evenly distributed over temperatures and grow systematically with pressure. At atmospheric pressure, both formulas agree with the current one for all temperatures within a 4 ppm error limit.

The Gibbs potential is completely determined now by specific volume, $v(S,t,p)$, and heat capacity, $c_P(S,t,0)$, except for enthalpy, $h(S,0,0)$, and entropy, $\sigma(S,0,0)$, of seawater at $t = 0$ °C and atmospheric pressure, $p = 0$ MPa, as,

$$g(S,t,p) = h(S,0,0) - \sigma(S,0,0)\cdot T - \int_0^t dt' \int_0^{t'} dt'' \frac{c_P(S,t'',0)}{T_0 + t''} + \int_0^p dp' v(S,t,p') \tag{46}$$

Sections 12–14 describe the computation of these two remaining unknown functions of salinity.

12. Limiting laws

At very low salt concentrations, the Debye-Hückel limiting laws govern the deviations of thermodynamic properties from those of pure water. Their exact measurement is technically difficult; however, they are required for proper fits of experimental data like freezing point depression or mixing heat (Bromley, 1968, Millero, Hansen & Hoff, 1973b) to the Gibbs potential. Fortunately, they can be derived sufficiently easily from the statistical theory of dilute mixed electrolytes (e.g. Lewis & Randall, 1961, Landau and Lifschitz, 1966, Falkenhagen and Ebeling, 1971). The following consideration briefly repeats the formulas given in FH95 except for a number of newer fundamental and molecular constants adopted here.

Up to terms $O(S^{3/2})$, the theoretical series expansion with respect to S of free enthalpy at atmospheric pressure takes the form (Landau and Lifschitz, 1966, Falkenhagen and Ebeling, 1971, Feistel and Hagen, 1995),

$$g(S,t,0) = c_{00} + c_{01}T + N_S kT\cdot S \ln S + (c_{20} + c_{21}T)\cdot S - \frac{kTv(t,0)}{12\pi}\cdot\kappa(S,t,0)^3 \tag{47}$$

Its several particular quantities are explained in the following. The Debye parameter κ (the reciprocal Debye radius of the ion cloud, not to be confused here with adiabatic compressibility) of seawater is given by,

$$\kappa(S,t,p)^2 = \frac{<Z^2>e^2}{\varepsilon(t,p)\varepsilon_0 kTv(t,p)}\cdot N_S\cdot S \tag{48}$$

All four free constants, c_{ij}, are to be determined e.g. by the choice of arbitrary reference states (Fofonoff, 1962, Feistel and Hagen, 1995). $c_{00} = h(0,0,0)$ is the enthalpy of pure water at atmospheric pressure and 0 °C, and $-c_{01} = \sigma(0,0,0)$ is its entropy. Both are determined by the conditions (Eqs. (33) and (34)) prescribed for the triple point. The other two, c_{20} and c_{21}, are to be fixed by a similar condition for standard seawater of salinity $S = 35$, as, $h(35,0,0) = 0$ and $\sigma(35,0,0) = 0$, after all other remaining coefficients of F03 have finally been evaluated. c_{20} and c_{21} are not required for the agreement with experimental data of seawater.

Eqs. (47) and (48) depend on the explicit details of the dissolved salt components only in compact form via two different quantities, the valence factor $<Z^2>$ and the particle number N_S. We are going to explicitly determine these two figures now from the molecular properties of the electrolytic solution. For the required stoichiometric definition of seasalt we use the specification given in Table 5 The mutual ratios of mass fractions W_a were taken from Millero (1982), except for sodium, which was derived from the electroneutrality condition using ion charges Z_a,

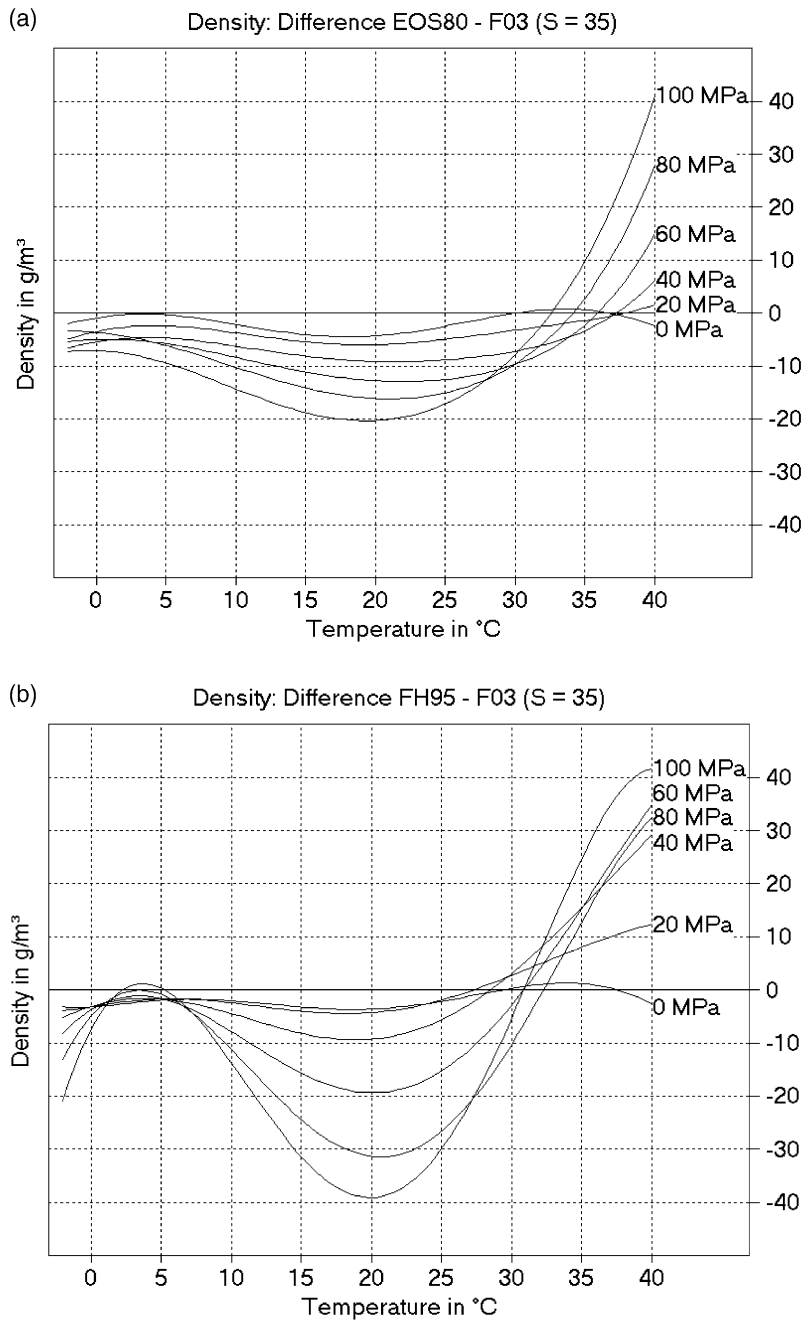

Fig. 26. Comparison of seawater densities, $\rho(S,t,p)$, at applied pressures p between 0 and 100 MPa (0 and 10,000 dbar) computed by (a) EOS80, above, and (b) FH95, below, with F03. FH95 high-pressure densities were derived from CM77 sound speeds for non-Neptunian waters and exhibit strong deviations from the direct density measurements used for F03.

Table 5
Average seawater composition and additional relevant atomic weights. Components and mass fractions from Millero (1982), mole fractions recomputed with IUPAC 99 mole masses (Coplen, 2001). Molecular weights are computed as sums over their atomic parts. *Z*: ion charge, *A*: atomic weight, *W*: weight fraction, *X*: mole fraction

kind (a)	Z_a	A_a /(g/mol)	W_a /ppm	X_a /ppm
Na	+1	22.989 77	306 563.3	418 791.8
Mg	+2	24.305	36 499.9	47 163.7
Ca	+2	40.078	11 716.8	9 181.5
K	+1	39.098 3	11 347.7	9 115.1
Sr	+2	87.62	225.9	81.0
Cl	−1	35.453	550 257.7	487 445.1
SO_4	−2	96.062 6	77 120.1	25 213.1
HCO_3	−1	61.016 84	3 228.0	1 661.5
Br	−1	79.904	1 911.5	751.3
CO_3	−2	60.008 9	330.5	173
$B(OH)_4$	−1	78.840 36	187.3	74.6
F	−1	18.998 403 2	33.1	54.6
H_3BO_3	0	61.833 02	578.4	293.8
H		1.007 94		
B		10.811		
C		12.010 7		
O		15.999 4		
S		32.065		
Ag		107.868 2		
H_2O		18.015 28		

$$\sum_a X_a \cdot Z_a = 0 \tag{49}$$

The mole fractions X_a were recomputed using IUPAC-99 mole masses (Coplen, 2001), A_a, by,

$$X_a = \frac{W_a/A_a}{\sum_b W_b/A_b} \tag{50}$$

We get for the average molar mass $<A> = \Sigma X_a \cdot A_a = 31.4060$ g/mol and for the valence factor $< Z^2> = \Sigma X_a \cdot Z_a{}^2 = 1.245143$.

N_S is the number of dissolved particles in 1 kg seawater with practical salinity $S = 1$. The link between both these descriptions of salinity is provided by chlorinity *Cl*, which is defined as the mass of silver needed to precipitate all chlorine and bromine in 328.5233 g of seawater (Millero and Leung, 1976b, Millero and Sohn, 1992),

$$Cl = 328.5233\ kg \cdot c \cdot A_{Ag} \cdot \left(\frac{W_{Cl}}{A_{Cl}} + \frac{W_{Br}}{A_{Br}} \right) \tag{51}$$

Here, *c* is the absolute salinity in grams of salt per kg of seawater. On the other hand, *Cl* can approximately related to practical salinity by the old conversion formula (Millero and Leung, 1976b, Lewis and Perkin, 1981),

$$S = 1.80655 \cdot Cl/\mathrm{g} \tag{52}$$

Eliminating *Cl*, we obtain the desired relation between practical and absolute salinity,

$$S = 1.004867 \cdot c \ / (\mathrm{g/kg}) \tag{53}$$

The values of the fundamental physical constants (Table 6) we have taken from the recent review of Mohr & Taylor (1999, p. 1808).

We can now determine the number of dissolved salt particles per practical salinity unit and per kg of seawater, as,

$$N_S = \frac{N_A \cdot c}{< A > \cdot S} = 1.926845 \cdot 10^{22} / \mathrm{kg} \tag{54}$$

Having determined the two constants $<Z^2>$ and N_S, we can return to the remaining unknowns of Eqs. (47) and (48). Both the relative dielectric constant $\varepsilon(t, p)$ of water at 0 °C and atmospheric pressure and its temperature derivative are obtained from the IAPWS (1997) release to be $\varepsilon(0,0) = 87.9034$, and $(\partial\varepsilon/\partial t)_p = -0.402570/\mathrm{K}$. The specific volume $v(t, p)$ of pure water at 0 °C and atmospheric pressure, $v(0,0) = 1.000157\ \mathrm{dm^3/kg}$, and its thermal expansion coefficient, $\alpha(0,0) = -67.7594\ \mathrm{ppm/K}$, are provided by the IAPWS95 equation of state.

Using these values in Eq. (47) after series expansion into powers of temperature around 0 °C up to linear terms, further 4 coefficients have been computed as given in the appendix.

13. Freezing point of seawater

The freezing temperature $t_f(S,p)$ of seawater with salinity S exposed to an applied pressure p can be calculated from the condition of thermodynamic equilibrium between seawater and ice,

$$\mu^W(S,t_f,p) = \mu^{Ice}(t_f,p) \tag{55}$$

which states that the chemical potentials of water must be the same in both phases.The chemical potential of water in seawater is (Feistel and Hagen, 1995, 1998),

$$\mu^W(S,t,p) = g(S,t,p) - S \cdot \left(\frac{\partial g}{\partial S}\right)_{t,p} \tag{56}$$

The chemical potential of ice is its specific free enthalpy,

$$\mu^{Ice}(t,p) = g^{Ice}(t,p) \tag{57}$$

A thermodynamic potential of pure water ice, $g^{Ice}(t,p)$, has been proposed by Feistel and Hagen (1995, 1998) for the vicinity of the freezing point, derived from various properties of ice. It has the simple polynomial form

Table 6
Recent values of fundamental physical constants after Mohr and Taylor (1999)

Name	Symbol	Value	Unit
Avogadro's constant	N_A	6.022 141 994 7 E+23	1/mol
molar gas constant	R	8.314 472 15	J/(mol K)
Boltzmann's constant	$k = R/N_A$	1.380 650 324 E−23	J/K
vacuum permittivity	ε_0	8.854 187 817 E−12	As/(Vm)
electron charge	e	1.602 176 462 63 E−19	As

$$g^{Ice}(t,p) = 1\ \mathrm{J/kg}\cdot\sum_{i,j} g_{ij}^{Ice}\cdot y^i\cdot z^j \tag{58}$$

with the variables $y = t/40°C$ and $z = p/100$ MPa, and coefficients as given in Table 7 The coefficients g_{00}^{Ice} and g_{10}^{Ice} have been differently determined for F03 in comparison to FH95 by adopting the IAPWS95 reference state, i.e. the condition that the chemical potentials of air-free liquid water and ice have to coincide at the triple point (T_t = 273.16 K, P_t = 611.657 Pa, Wagner and Pruß, 2002),

$$g^{IAPWS95}(T_t,P_t) = g^{Ice}(t_t,p_t) \tag{59}$$

and that enthalpies of liquid water and ice differ by melting heat h^{melt} = 333.5 kJ/kg (Rossini, Wagman, Evans, Levine, & Jaffe 1952) at freezing temperature t_f = 0.0026 °C under atmospheric pressure p_f = 0 MPa.

$$h^{IAPWS95}(T_f,P_f) = h^{Ice}(t_f,p_f) + h^{melt} \tag{60}$$

Introducing now temporarily a 2nd order relative free enthalpy of ice, η, by,

$$\eta(t,p) = g^{Ice}(t,p) - (g_{00}^{Ice} + g_{10}^{Ice}\cdot t/40°C)\cdot 1\ \mathrm{J/kg} \tag{61}$$

we can formally solve Eqs. (59) and (60) for the unknown coefficients, as,

$$g_{00}^{Ice} = \frac{T_0\cdot\Delta g_t + t_t\cdot\Delta h_f}{T_t} \tag{62}$$

$$g_{10}^{Ice} = \frac{40°C\cdot(\Delta g_t - \Delta h_f)}{T_t} \tag{63}$$

Here, the difference terms are abbreviations,

$$\Delta g_t\cdot 1\ \mathrm{J/kg} = g^{IAPWS95}(T_t,P_t) - \eta(t_t,p_t) \tag{64}$$

$$\Delta h_f\cdot 1\ \mathrm{J/kg} = h^{IAPWS95}(T_f,P_f) - h^{melt} - \eta(t_f,p_f) + T_f\cdot\frac{\partial\eta(t_f,p_f)}{\partial t} \tag{65}$$

The results are given in Table 7 A cubic high-pressure correction g_{03}^{Ice} additional to FH95 has been obtained by fitting Eq. (55) to IAPWS95 melting pressures up to 50 MPa, which are based on high-pressure Henderson & Speedy (1987) data with accuracy 100 mK.

The freezing temperature resulting from Eq. (55) of pure water at atmospheric pressure is t_f = 0.002 5 °C, which is in good agreement with t_f = 0.002 60 °C of IAPWS95, and with the freezing point depression of 2.4 mK to 0 °C due to saturation with air (Doherty & Kester, 1974). The lowering is slightly less (1.9 mK) for seawater, such that a corresponding small correction to the freezing point measurements of Doherty and Kester (1974) can be estimated as,

Table 7
Coefficients of free enthalpy of ice, Eq. (58)

g_{ij}^{Ice}	y^0	y^1	y^2	y^3
z^0	98.266 90	48 842.28	−6 139.045	21.782 64
z^1	109 085.8	690.076 5	60.652 68	
z^2	−1 266.486	−99.207 47		
z^3	363.422 5			

$$t_f^{air-free} = t_f^{DK74} + 2.4\ \mathrm{mK} - \frac{S}{35}\cdot 0.5\ \mathrm{mK} \tag{66}$$

for air-free water conditions assumed in the current paper.

Wagner, Saul & Pruß (1994) provide freezing temperatures of pure water for a wide range of pressures (see also Wagner and Pruß, 2002). Fig. 27 shows several freezing point formulas in comparison to them. The low-pressure range is probably less precisely described by Wagner et al. (1994) (about 6 mK, see Fig. 27) than by Millero (1978) or Feistel and Hagen (1995, 1998), due to the use of more recent ice density data by the latter two. Both were derived for air-saturated water, causing the intercept of 2.6 mK at atmospheric pressure in the drawing. The seawater formulas of Doherty and Kester (1974) and Fujino, Lewis & Perkin (1974) are shown for $S = 0$ but are not claimed valid for pure water by their authors. The formula of Millero (1978) is assumed to hold within an error of 3 mK up to 5 MPa. If we accept a tolerance of about 10 mK, Fig. 27 shows that FH95 freezing points are likely to be valid even beyond 30 MPa (or 3000 meters of water depth), and the current improved ice potential function (Table 7) extends this range to about 50 MPa (5000 dbar), thus becoming applicable to cases like Antarctic subglacial lakes (Siegert et al., 2001). As an alternative to the Gibbs potential of ice proposed here (Eq. (58)), the one derived by Tillner-Roth (1998) can be used for very high pressures. It is derived from ice properties as well as from the melting pressure curve of Wagner et al. (1994), is valid even beyond 100 MPa, and reproduces the freezing point measurements of Henderson and Speedy (1987) within 100 mK. With only 2 mK deviation up to $p = 5$ MPa, it is in excellent agreement with F03 at low pressures (Fig. 27). Its error in case of application to seawater can be assumed to be of the same magnitude.

As follows from Eq. (46), only measurements at atmospheric pressure are required to determine the unknown parts of the thermodynamic potential function. High-pressure freezing points can be derived then

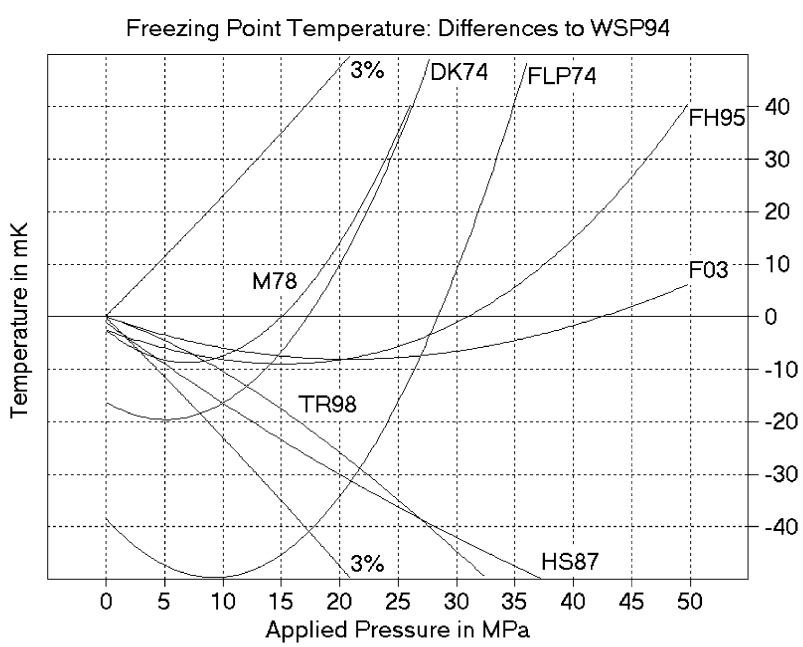

Fig. 27. Comparison of freezing point temperature formulas for pure water, $t_f(0,p)$, with WSP94 (Wagner et al., 1994). DK74: Doherty and Kester (1974), FLP74: Fujino et al. (1974), M78: Millero (1978), HS87: Henderson and Speedy (1987), TR98: Tillner-Roth (1998), FH95: Feistel and Hagen (1995, 1998), F03: this paper. The 3% cone indicates the uncertainty of the WSP94 melting pressure function, corresponding to 3% of the melting pressure.

immediately from Eqs. (46) and (55), limited in pressure mainly by the validity of the chemical potential function of water in ice, Eq. (66).

Freezing point measurements (S_f, t_f) at ambient pressure of Doherty and Kester (1974) were used for a joint regression together with mixing heats, as described in section 14. The expression

$$\sum\left\{g(S_f,t_f,0)-S_f\cdot\left(\frac{\partial g}{\partial S}\right)_{t,p=0}-g^{Ice}(t_f,0)\right\}^2 = Min \tag{67}$$

was minimized at 32 data points after correcting temperatures due to Eq. (66), with a required tolerance of 2 J/kg in free enthalpy. 8 coefficients g_{ij0}, $i = 4\ldots7$, $j = 0,1$ have been calculated as given in the appendix. The resulting r.m.s. of the fit was 1.8 J/kg, corresponding to 1.5 mK mean deviation in freezing temperatures. The scatter of the data is shown in Fig. 28, together with the one-atmosphere measurements of Fujino et al. (1974). The latter ones were not corrected for dissolved air, a tentative application of Eq. (66) to these data increases the mean error from 2.6 to 4.3 mK.

14. Mixing heat of seawater

Bromley (1968) reported seawater mixing experiments at $t = 25°C$ (IPTS-48 scale assumed) with accuracy estimated by the author to 1 cal. Out of these, 24 samples with initial masses m_1, m_2 and salinities S_1, $S_2 < 34$ were used with their heat effect Q after mixing, converted by 4.1840 J/cal. We have minimized the sum (Feistel and Hagen, 1995),

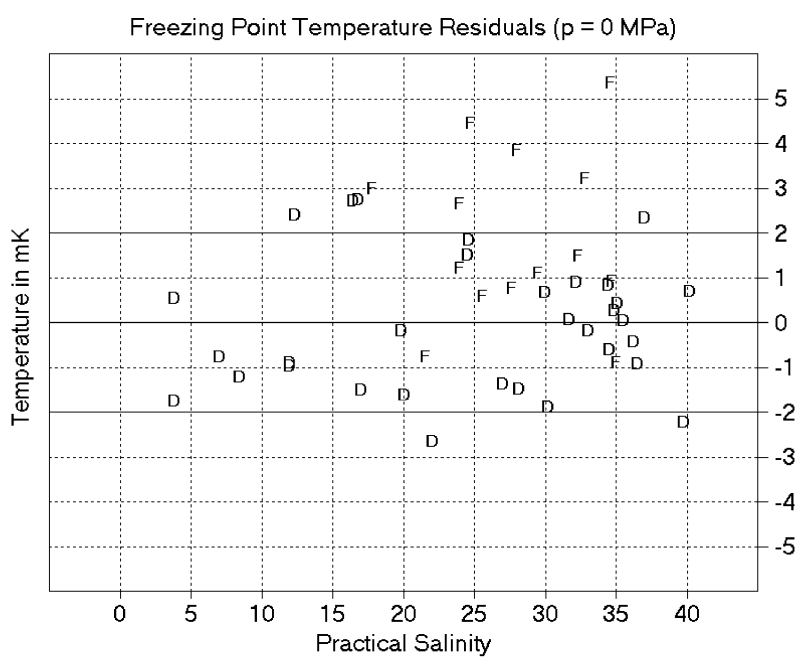

Fig. 28. Deviation of measured freezing points of seawater, $t_f(S,0)$, at atmospheric pressure ($P = 0.101325$ MPa) from temperatures computed by Eq. (55). D: Doherty and Kester (1974) with air correction (56), F: Fujino et al. (1974) without air correction. Horizontal lines mark the experimental uncertainty.

$$\sum\left\{(m_1 + m_2)\cdot h(\frac{m_1S_1 + m_2S_2}{m_1 + m_2},t,0) - m_1h(S_1,t,0) - m_2h(S_2,t,0) + Q\right\}^2 = Min \tag{68}$$

over these measurements with required r.m.s. of 4 J, in conjunction with freezing point measurements (section 13) and dilution heats (below). Enthalpies h are expressed by the potential function $g(S,t,p)$ as,

$$h(S,t,p) = g(S,t,p) - (T_0 + t)\cdot\left(\frac{\partial g}{\partial t}\right)_{S,p} \tag{69}$$

The regression resulted in an r.m.s. deviation of 2.4 J, as shown in Fig. 29(a).

Millero et al. (1973b) performed experiments diluting seawater from salinities S_1 to S_2 at various temperatures t (assumed IPTS-48 scale), reporting relative enthalpies L with accuracy of 5 cal/eq, as given in that paper. We have minimized the sum (Feistel and Hagen, 1995) over 95 of their data points $L(S_1, S_2, t)$, converted from cal/eq to J/kg multiplying by 4.184/57.754,

$$\sum\left\{\frac{h(S_2,t,0) - h(0,t,0)}{S_2} - \frac{h(S_1,t,0) - h(0,t,0)}{S_1} + L\right\}^2 = Min \tag{70}$$

with required r.m.s. of 0.4 J/kg, together with Bromley's mixing heats and freezing points of Doherty et al. The resulting scatter of 0.5 J/kg is displayed in Fig. 29(b).

Shapes of resulting excess free enthalpies, $g(S,t,0) - g(0,t,0)$, entropies, $\sigma(S,t,0) - \sigma(0,t,0)$, and enthalpies, $h(S,t,0) - h(0,t,0)$, relative to pure water and at atmospheric pressure, are shown in Fig. 30.

Millero and Leung (1976b) and Millero (1983) have published formulas for relative specific entropies $\Delta\sigma^{ML76} = \sigma(S,t,0) - \sigma(0,t,0)$, enthalpies $\Delta h^{ML76} = h(S,t,0) - h(0,t,0)$ and free enthalpies $\Delta g^{ML76} = g(S,t,0) - g(0,t,0)$ of seawater at one atmosphere. Their salinity dependencies at $t = 0$ °C show a good agreement with the corresponding expressions derived here in sections 12–14, as shown in Fig. 30. Error estimates of those authors were 2 J/kg for free enthalpy, 0.1 J/(kg K) for entropy and 3 J/kg for enthalpy. The r.m.s. deviations visible in Fig. 30 are in the order of 10%: 21 J/kg in free enthalpy, 0.08 J/(kg K) in entropy, and 9 J/kg in enthalpy.

Before both functions could be compared they had to be aligned to a common reference state by suitably adjusting the gauge constants A, B (compare Eq. (47)),

$$g(S,t,0) - g(0,t,0) = \Delta g^{ML76}(S,t) + (A + B\cdot t)\cdot S \tag{71}$$

$$g(S,t,0) - g(0,t,0) - T\frac{\partial[g(S,t,0) - g(0,t,0)]}{\partial t} = \Delta h^{ML76}(S,t) + (A - B\cdot T_0)\cdot S \tag{72}$$

$$-\frac{\partial[g(S,t,0) - g(0,t,0)]}{\partial t} = \Delta\sigma^{ML76}(S,t) - B\cdot S \tag{73}$$

Requiring $g(35, 0, 0) = 0$ and $h(35, 0, 0) = 0$ we derive A, B from Eqs. (71), (72) to be,

$$A = -\frac{1}{35}[g(0,0,0) + \Delta g^{ML76}(35,0)] = 139.9\ \mathrm{J/kg} \tag{74}$$

$$B = -\frac{1}{35T_0}\left[T_0\frac{\partial g(0,0,0)}{\partial t} + \Delta g^{ML76}(35,0) - \Delta h^{ML76}(35,0)\right] = 0.4867\ \mathrm{J/(kg\ K)} \tag{75}$$

Temperature dependencies of the functions given by Millero and Leung (1976b), however, do not obey the thermodynamic relations $\Delta c_p = T(\partial\Delta\sigma/\partial t) = \partial\Delta h/\partial t = -T(\partial^2\Delta g/\partial t^2)$ with relative specific heat capacities, $\Delta c_p = c_p(S,t,0) - c_p(0,t,0) \leq 0$. Rather, they are related to relative partial molal heat capacities,

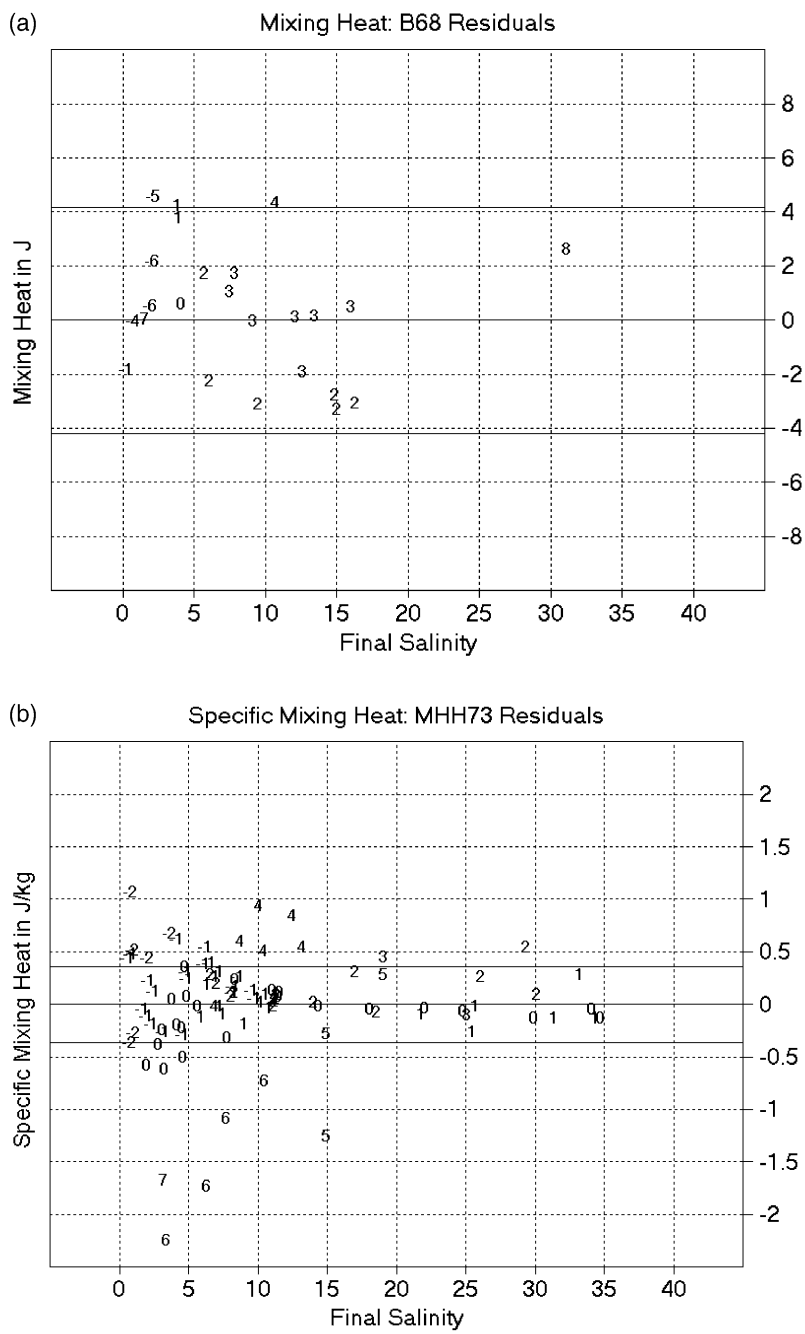

Fig. 29. Deviations of measured mixing heats, plotted vs. final salinities S of the mixtures. Experimental uncertainty is indicated by horizontal lines. (a) Bromley (1968), above, after Eq. (68). Symbols are values of Q in J, rounded to integer. (b) Millero et al. (1973b), below, after Eq. (70). Symbols are values of $L/2$ in J/kg, rounded to integer.

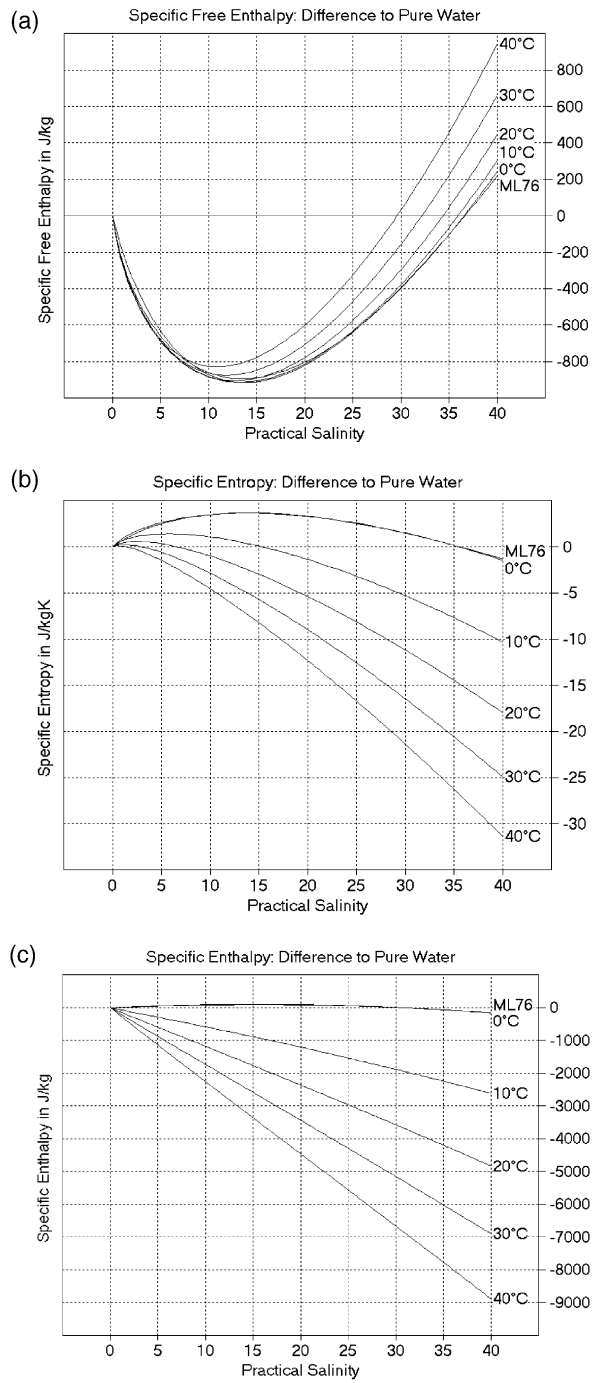

Fig. 30. (a) above, relative specific free enthalpy, $g(S,t,0)-g(0,t,0)$, (b) middle, relative specific entropy, $\sigma(S,t,0)-\sigma(0,t,0)$, and (c) below, relative specific enthalpy, $h(S,t,0)-h(0,t,0)$, as functions of salinity S and temperature t at ambient pressure $p = 0$ MPa. ML76: corresponding functions by Millero and Leung (1976b) at $t = 0$ °C after alignment to a common reference state due to Eqs. (71)–(73).

$\Phi_{cp}-\Phi^0_{cp}$, as defined in their paper (Millero, 2003). Hence, they cannot be compared directly with the F03 functions derived here except for $t = 0$ °C.

15. Conclusion

With the compilation of the current F03 a final state of the longer development of the Gibbs thermodynamic potential of seawater is reached, mainly marked by the now extensive involvement of experimental data together with the new pure water standard IAPWS95. It has significantly improved and extended its predecessor versions, as is summarized in Tables 8 and 9.

Modern quantitative descriptions of thermodynamic seawater properties include fresh water in the limiting case of vanishing salinity. This holds for the International Equation of State of Seawater 1980 and its accompanying equations for isobaric specific heat capacity and sound speed. EOS80 had been derived by

Table 8
Symbolic scheme of background papers used for subsequent comprehensive formulations of seawater equilibrium thermodynamics: EOS80 (Millero, Perron and Desnoyers, 1973a, Chen and Millero, 1977, Millero et al., 1980, Millero and Poisson, 1981a), F93 (Feistel, 1993), FH95 (Feistel and Hagen, 1995), F03 (this paper). This simplified list of implicit or explicit sources or links is neither unique nor complete

Source	EOS80	F93	FH95	F03
IAPWS95				$g(0,t,p)$
PG93				$v(S,t,0)$
BS86	$v(S,t,p)$			
EOS80		$v(S,t,0)$	$v(S,t,0)$	
PBB80	$v(S,t,0)$			$v(S,t,0)$
C78			$\alpha(S,t,p)$	$\alpha(S,t,p)$
CM77	$U(S,t,p)$	$v(S,t,p)$	$v(S,t,p)$	
BS76	$v(S,t,p)$			
CM76	$v(S,t,p)$			$v(S,t,p)$
MGW76	$v(S,t,0)$			$v(S,t,0)$
ML76		$g(S,t,0)$		
DG74			$v(S,t,p)$	$v(S,t,p)$
DK74			$g(S,t,0)$	$g(S,t,0)$
MPD73	$c_P(S,t,0)$	$c_P(S,t,0)$	$c_P(S,t,0)$	$c_P(S,t,0)$
MHH73			$h(S,t,0)$	$h(S,t,0)$
DGM72	$U(0,t,0)$			
DG70	$U(S,t,0)$			
BDSW70				$c_P(S,t,0)$
BS70	$v(S,t,p)$			$v(S,t,p)$
B68			$h(S,t,0)$	$h(S,t,0)$
B67	$v(0,t,0)$			
W59	$U(0,t,p)$			
S55	$c_P(0,t,0)$			

Sources are IAPWS95 (Wagner and Pruß, 2002), PG93 (Poisson and Gadhoumi, 1993), BS86 (Bradshaw and Schleicher, 1986), EOS80 (Fofonoff and Millard, 1983), PBB80 (Poisson et al., 1980), C78 (Caldwell, 1978), CM77 (Chen and Millero, 1977), BS76 (Bradshaw and Schleicher, 1976), CM76 (Chen and Millero, 1976), MGW76 (Millero et al., 1976a), ML76 (Millero and Leung, 1976b), DG74 (Del Grosso, 1974), DK74 (Doherty and Kester, 1974), MPD73 (Millero et al., 1973a), MHH73 (Millero et al., 1973b), DGM72 (Del Grosso and Mader, 1972), DG70 (Del Grosso, 1970), BDSW70 (Bromley et al., 1970), BS70 (Bradshaw and Schleicher, 1970), B68 (Bromley, 1968), B67 (Bigg, 1967), W59 (Wilson, 1959), S55 (Stimson, 1955).Quantities are S (salinity), t (temperature), p (pressure), g (specific free enthalpy), v (specific volume), α (isobaric thermal expansion coefficient), U (sound speed), c_P (specific heat at constant pressure), h (specific enthalpy).

Table 9
Summary of data used for the determination of F03 free enthalpy polynomial coefficients by regression. "Required" is the r.m.s. prescribed in the least-square condition of the particular data set, "resulting" is the returned r.m.s. of the fit. Column "s." refers to the section where further details are described. IAPWS95: Wagner and Pruß (2002), MGW76: Millero et al. (1976), PBB80: Poisson et al. (1980), PG93: Poisson and Gadhoumi (1993), BDSW70: Bromley et al. (1970), MPD73: Millero et al. (1973a), C78: Caldwell (1978), I–VI: Del Grosso (1974), CM76: Chen and Miller (1976), BS70: Bradshaw and Schleicher (1970), DK74: Doherty and Kester (1974), B68: Bromley (1968), MHH73: Millero et al. (1973b)

Quantity	Source	S	t /°C	p /MPa	Points	Required r.m.s.	Resulting r.m.s.	s.
$c_P(0,t,0)$	IAPWS95	0	0–45	0	8	10 J/(kg K)	0.01 J/(kg K)	2
$\alpha(0,t,0)$	IAPWS95	0	0–45	0	8	0.6 ppm/K	0.005 ppm/K	3
$\alpha(0,t,p)$	IAPWS95	0	−7–45	−0.1–110	64	0.6 ppm/K	0.01 ppm/K	3
$\rho(0,t,0)$	IAPWS95	0	0–45	0	8	1 ppm	0.032 ppm	4
$\rho(0,t,p)$	IAPWS95	0	−7–45	−0.1–110	64	30 ppm	0.020 ppm	4
$K(0,t,p)$	IAPWS95	0	−7–45	−0.1–110	64	0.4 ppm/MPa	0.003 ppm/MPa	5
$\rho(S,t,0)$	MGW76	0.5–40	0–40	0	122	4 ppm	4.1 ppm	7
$\rho(S,t,0)$	PBB80	5–42	0–30	0	345	4 ppm	4.0 ppm	7
$\rho(S,t,0)$	PG93	34–50	15–30	0	81	10 ppm	11.3 ppm	7
$c_P(S,t,0)$	BDSW70	10–50	0–40	0	25	2 J/(kg K)	0.54 J/(kg K)	8
$c_P(S,t,0)$	MPD73	1–40	5–35	0	48	0.5 J/(kg K)	0.52 J/(kg K)	8
$\alpha(S,t,p)$	C78	10–30	−6–1	0.7–33	31	0.6 ppm/K	0.73 ppm/K	9
$U(S,t,p)$	I-III	29–43	0–35	0–2	92	5 cm/s	1.7 cm/s	10
$U(S,t,p)$	IVa-d	29–43	0–30	0.1–5	32	5 cm/s	1.2 cm/s	10
$U(S,t,p)$	V-VI	33–37	0–5	0–100	128	5 cm/s	3.5 cm/s	10
$v(S,t,p)$	CM76	5–40	0–40	0–100	558	10 ppm	11.0 ppm	11
$v(S,t,p)$	BS70	30–40	−2–30	1–100	221	4 ppm	2.6 ppm	11
$g(S,t,0)$	DK74	4–40	−2–0	0	32	2 J/kg	1.8 J/kg	13
$Q(S,t,0)$	B68	0–33	25	0	24	4 J	2.4 J	14
$L(S,t,0)$	MHH73	1–41	0–30	0	95	0.4 J/kg	0.5 J/kg	14

combining various extended and precise fundamental measurements of seawater properties, many of them were calibrated with, or made relative to, or included as special case, corresponding data of pure water. The dependence of these seawater data on their pure water reference was intentionally made explicit by their authors in some cases, like for heat capacities by Millero et al. (1973a), or specific volumes by Chen et al. (1976), but in other cases it is hardly reproducible afterwards, like for heat capacities by Bromley et al. (1970).

The first Gibbs function F93 was an attempt towards a more consistent, comprehensive and extended formulation, compared to the EOS80 triple. It was a combination of EOS80 density and heat capacity at ambient pressure, as well as EOS80 sound speed for their extrapolation to higher pressures. The resulting PVT properties of the potential function were compiled to remain as numerically close as possible to the EOS80 original, and were completed with the thermochemical properties of ML76, published by Millero and Leung (1976b).

The subsequent version FH95 of the Gibbs potential was developed to allow for mainly three small but important corrections with respect to F93.

1. Sound speed of Chen and Millero (1977), CM77, had been found to exhibit systematic errors of about 0.5 m/s in deep-water travel time measurements (Dushaw et al., 1993, Millero and Li, 1994) and needed to be partially replaced by the more accurate function of Del Grosso (1974), DG74, which is valid for Neptunian waters only.
2. Temperatures of maximum density computed by EOS80 severely deviate from the measurements of Caldwell (1978) especially for brackish waters, such that these measurements needed to be involved.

3. The limiting laws of F93 included terms in the power expansion with respect to salinity which were inconsistent with electrolyte theory. The corresponding formulas used in F93 were substituted by the correct Debye-Hückel limiting law and completed with experimental data for freezing point temperatures of Doherty and Kester (1974), and mixing enthalpies of Bromley (1968) and Millero et al. (1973b), replacing the ML76 functions used for F93.

With the release of the IAPWS95 formulation (IAPWS, 1996) and its detailed description by Wagner and Pruß (2002), a comprehensive high-precision standard for equilibrium thermodynamic properties of Vienna Standard Mean Ocean Water (Gonfiantini, 1978) has become available. Like its precursor, the IAPS-84 standard (Haar, Gallagher & Kell, 1988), it is expressed by specific free energy $f(T,\rho)$ of the fluid phases as function of absolute temperature and density over a wide range of conditions, including those of oceanic and limnic waters. It has triggered the revision of the seawater potential function, involving the substitution of all explicit or implicit references to old pure water data by the new standard. It further suggested the idea that the still existing inconsistency between the sound speed formulas CM77 and DG74 could be resolved in a natural way, which is likely caused by improper pure water calibration (Millero and Li, 1994). It was therefore necessary to recalibrate the experimental data which served for the construction of EOS80, and to use them directly for a new determination of the polynomial coefficients declared by Eq. (1). As an example, old heat capacity data from 1902–1927, summarized in the paper of Stimson (1955), were used to determine seawater specific heat capacities of seawater by Millero et al. (1973a), thus became part of EOS80 and were finally passed through to F93 and FH95 this way (Table 8).

The current Gibbs potential F03 is no longer based on any of the three EOS80 functions which lead to the previous versions F93 and FH95. The background data used for EOS80 and for the new formulation are partly the same, compare Table 8, nonetheless both can be considered as independently derived analytical expressions representing the particular experimental data.

F03 is completely consistent with the most accurate and comprehensive description of pure water available. Its zero salinity part is valid from the triple point to 110 MPa applied pressure and from the freezing temperature to 45 °C. A tentative recalibration of CM77 sound speeds with respect to IAPWS95 turned out to be insufficient to explain the differences to DG74. On the other hand, specific volumes measured by Bradshaw and Schleicher (1970) and Chen and Millero (1976) after recalibration appeared fairly well consistent with DG74 sound speeds, and have been involved as preferred substitutes for CM77 in the current regression procedure. They are even slightly better reproduced now than originally by EOS80, compare Figs. 23 and 24. Densities and heat capacities at atmospheric pressure have been adjusted for the new IAPWS95 reference and extended up to salinities 50 by the data of Bromley et al. (1970) and Poisson and Gadhoumi (1993). Freezing points computed with F03 have been made compatible with IAPWS95 by correcting for dissolved air, and are extended in validity to higher pressures by an improved Gibbs potential of ice. The reference state of IAPWS95 (zero internal energy and entropy of the liquid phase at the triple point of pure water) has been adopted for the F03 Gibbs potential of ice, in distinction to FH95.

Table 9 shows a summary of all data used in four different regression computations of this paper, together with their required and achieved precisions. The new formulation F03 reproduces the measurements of Millero et al. (1976a) within 4.1 ppm, Poisson et al. (1980) within 4.0 ppm, Poisson and Gadhoumi (1993) within 11.3 ppm, Bromley et al. (1970) within 0.54 J/(kg K), Millero et al. (1973a) within 0.52 J/(kg K), Caldwell (1978) within 0.73 ppm/K, Del Grosso (1974) within 3.5 cm/s, Chen and Millero (1976) within 11.0 ppm, Bradshaw and Schleicher (1970) within 2.6 ppm, Doherty and Kester (1974) within 1.5 mK, Bromley (1968) within 2.4 J and Millero et al. (1973b) within 0.5 J/kg. 216 points from IAPWS95 functions were used for numerical integrals to fit 39 coefficients of pure water, 1834 data points were used to adjust 54 coefficients of seawater properties of the current thermodynamic potential function to various measurements. Additionally, 2 coefficients of water and 6 of seawater have been calculated from reference states and limiting laws. Compared to FH95, the number of polynomial coefficients has increased from 89

to 101. Of the additional 12 terms, 9 were required for pure water to ensure consistency with IAPWS95, and further 2 were added for an improved description of freezing points and dilution heats.

The new Gibbs thermodynamic potential function F03 is equivalent to the international pure water standard IAPWS95 in the oceanic ranges of temperatures and pressures. It represents a suitable compromise between various recalibrated seawater measurements within their corresponding experimental errors as shown in Table 9. This is of special importance for the high-pressure seawater data of Bradshaw and Schleicher (1970), Del Grosso (1974), Chen and Millero (1976) and Caldwell (1978). On one hand, the recalibration has systematically changed almost all seawater properties, some of them even significantly like sound speed at high pressure and temperature. On the other hand, changes compared to FH95 in the range of Neptunian waters are perceptible but hardly exceed the experimental tolerances. F03 can be considered more accurate and more reliable than its predecessors.

Still there are, however, carefully prepared seawater experiments which are not adequately reproduced by the actual formulation. One of them is sound speed of Chen and Millero (1977) as discussed in section 10. Further, the high-salinity extension by Poisson and Gadhoumi (1993) scatters much stronger than predicted by the authors (Section 7). Another case is lapse rates measured by Caldwell and Eide (1980) with claimed accuracy of 0.2%, which deviate from the ones computed here by up to 2%. Note that the latter value corresponds to the error estimate given by Bryden (1973), see the discussion by Fofonoff (1985).

In future versions, seawater with salinity higher than 50 at temperatures up to 500 °C and pressures up to 30 MPa will need to be covered in order to be applicable to processes like those in thermal desalination plants (Slesarenko & Shtim, 1989), or hydrothermal vents found on ocean ridges (Jupp & Schultz, 2000).

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Appendix

Tables 10–27

Table 10
Coefficients g_{ijk} of specific free enthalpy $g(S,t,p)$ with variables $S = 40 \cdot x^2$, temperature $t = 40\ °C \cdot y$, and applied pressure $p = 100$ MPa·z:

$$g(S,t,p) = 1\ \mathrm{J/kg} \cdot \sum_{j,k} \left\{ g_{0jk} + g_{1jk} x^2 \ln x + \sum_{i>1} g_{ijk} x^i \right\} y^j z^k$$

i	j	k	g_{ijk}	i	j	k	g_{ijk}	i	j	k	g_{ijk}
0	0	0	101.342743139672	0	5	4	6.48190668077221	2	4	2	74.726141138756
0	0	1	100015.695367145	0	6	0	−18.9843846514172	2	4	3	−36.4872919001588
0	0	2	−2544.5765420363	0	6	1	63.5113936641785	2	5	0	−17.43743842213
0	0	3	284.517778446287	0	6	2	−22.2897317140459	3	0	0	−2432.0947227047
0	0	4	−33.3146754253611	0	6	3	8.17060541818112	3	0	1	199.459603073901
0	0	5	4.20263108803084	0	7	0	3.05081646487967	3	0	2	−52.2940909281335
0	0	6	−0.546428511471039	0	7	1	−9.63108119393062	3	0	3	68.0444942726459
0	1	0	5.90578348518236	1	0	0	5813.28667992895	3	0	4	−3.41251932441282
0	1	1	−270.983805184062	1	1	0	851.295871122672	3	1	0	−493.512590658728
0	1	2	776.153611613101	2	0	0	1376.28030233939	3	1	1	−175.292041186547
0	1	3	−196.51255088122	2	0	1	−3310.49154044839	3	1	2	83.1923927801819
0	1	4	28.9796526294175	2	0	2	384.794152978599	3	1	3	−29.483064349429
0	1	5	−2.13290083518327	2	0	3	−96.5324320107458	3	2	0	−158.720177628421
0	2	0	−12357.785933039	2	0	4	15.8408172766824	3	2	1	383.058066002476
0	2	1	1455.0364540468	2	0	5	−2.62480156590992	3	2	2	−54.1917262517112
0	2	2	−756.558385769359	2	1	0	140.576997717291	3	2	3	25.6398487389914
0	2	3	273.479662323528	2	1	1	729.116529735046	3	3	0	67.5232147262047
0	2	4	−55.5604063817218	2	1	2	−343.956902961561	3	3	1	−460.319931801257
0	2	5	4.34420671917197	2	1	3	124.687671116248	3	4	0	−16.8901274896506
0	3	0	736.741204151612	2	1	4	−31.656964386073	3	4	1	234.565187611355
0	3	1	−672.50778314507	2	1	5	7.04658803315449	4	0	0	2630.93863474177
0	3	2	499.360390819152	2	2	0	929.460016974089	4	0	1	−54.7919133532887
0	3	3	−239.545330654412	2	2	1	−860.764303783977	4	0	2	−4.08193978912261
0	3	4	48.8012518593872	2	2	2	337.409530269367	4	0	3	−30.1755111971161
0	3	5	−1.66307106208905	2	2	3	−178.314556207638	4	1	0	845.15825213234
0	4	0	−148.185936433658	2	2	4	44.2040358308	4	1	1	−22.6683558512829
0	4	1	397.968445406972	2	2	5	−7.92001547211682	5	0	0	−2559.89065469719
0	4	2	−301.815380621876	2	3	0	−260.427286048143	5	0	1	36.0284195611086
0	4	3	152.196371733841	2	3	1	694.244814133268	5	1	0	−810.552561548477
0	4	4	−26.3748377232802	2	3	2	−204.889641964903	6	0	0	1695.91780114244
0	5	0	58.0259125842571	2	3	3	113.561697840594	6	1	0	506.103588839417
0	5	1	−194.618310617595	2	3	4	−11.1282734326413	7	0	0	−466.680815621115
0	5	2	120.520654902025	2	4	0	97.1562727658403	7	1	0	−129.049444012372
0	5	3	−55.2723052340152	2	4	1	−297.728741987187				

Table 11
Specific free enthalpy, $g(S, t, p)$, in kJ/kg

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	0.101	−0.560	−0.759	−0.786	−0.702	−0.533	−0.295	0.000	0.346
0	5	−0.090	−0.762	−0.964	−0.991	−0.903	−0.729	−0.485	−0.181	0.174
0	10	−0.659	−1.339	−1.541	−1.564	−1.471	−1.289	−1.035	−0.721	−0.354
0	15	−1.598	−2.284	−2.483	−2.500	−2.398	−2.206	−1.941	−1.613	−1.232
0	20	−2.901	−3.590	−3.783	−3.792	−3.679	−3.474	−3.195	−2.852	−2.454
0	25	−4.560	−5.250	−5.436	−5.433	−5.307	−5.087	−4.792	−4.431	−4.015
0	30	−6.571	−7.259	−7.434	−7.418	−7.276	−7.040	−6.726	−6.346	−5.909
0	35	−8.926	−9.610	−9.772	−9.741	−9.582	−9.327	−8.992	−8.591	−8.131
0	40	−11.620	−12.297	−12.446	−12.397	−12.219	−11.942	−11.585	−11.160	−10.676
10	0	10.078	9.377	9.138	9.071	9.117	9.248	9.447	9.704	10.012
10	5	9.886	9.175	8.934	8.869	8.919	9.055	9.261	9.528	9.846
10	10	9.321	8.601	8.362	8.301	8.357	8.501	8.718	8.995	9.326
10	15	8.388	7.663	7.427	7.373	7.438	7.593	7.822	8.113	8.458
10	20	7.094	6.367	6.137	6.092	6.168	6.336	6.579	6.886	7.248
10	25	5.447	4.719	4.497	4.463	4.553	4.736	4.996	5.320	5.701
10	30	3.451	2.726	2.514	2.493	2.598	2.799	3.077	3.421	3.823
10	35	1.112	0.391	0.192	0.187	0.310	0.529	0.828	1.194	1.618
10	40	−1.564	−2.278	−2.463	−2.451	−2.308	−2.068	−1.747	−1.357	−0.908
20	0	20.005	19.265	18.987	18.882	18.890	18.983	19.145	19.365	19.636
20	5	19.815	19.065	18.786	18.683	18.696	18.795	18.964	19.194	19.476
20	10	19.254	18.496	18.219	18.121	18.141	18.248	18.429	18.670	18.964
20	15	18.328	17.566	17.293	17.202	17.231	17.350	17.543	17.798	18.107
20	20	17.045	16.281	16.014	15.932	15.972	16.105	16.312	16.583	16.910
20	25	15.410	14.646	14.387	14.317	14.371	14.519	14.743	15.032	15.377
20	30	13.428	12.667	12.419	12.362	12.432	12.597	12.839	13.148	13.515
20	35	11.107	10.349	10.114	10.073	10.160	10.344	10.608	10.938	11.328
20	40	8.450	7.699	7.477	7.454	7.561	7.766	8.052	8.407	8.821
40	0	39.718	38.901	38.549	38.370	38.304	38.323	38.412	38.560	38.760
40	5	39.535	38.710	38.357	38.181	38.121	38.148	38.246	38.404	38.615
40	10	38.986	38.155	37.805	37.635	37.582	37.619	37.728	37.899	38.123
40	15	38.078	37.243	36.898	36.736	36.693	36.742	36.864	37.049	37.289
40	20	36.817	35.981	35.643	35.490	35.460	35.522	35.659	35.861	36.118
40	25	35.209	34.373	34.043	33.903	33.886	33.964	34.119	34.339	34.616
40	30	33.258	32.425	32.106	31.979	31.979	32.074	32.248	32.488	32.786
40	35	30.970	30.142	29.835	29.725	29.742	29.857	30.051	30.313	30.634
40	40	28.351	27.528	27.237	27.143	27.181	27.317	27.534	27.820	28.166
100	0	97.827	96.798	96.237	95.851	95.579	95.393	95.279	95.224	95.222
100	5	97.692	96.657	96.099	95.718	95.454	95.279	95.175	95.133	95.143
100	10	97.204	96.165	95.611	95.238	94.984	94.820	94.729	94.700	94.726
100	15	96.368	95.327	94.779	94.415	94.172	94.021	93.944	93.932	93.974
100	20	95.188	94.147	93.606	93.253	93.024	92.887	92.827	92.831	92.892
100	25	93.670	92.630	92.099	91.759	91.544	91.424	91.382	91.405	91.486
100	30	91.819	90.782	90.263	89.937	89.738	89.636	89.613	89.657	89.759
100	35	89.639	88.607	88.100	87.790	87.610	87.528	87.526	87.592	87.718
100	40	87.136	86.109	85.617	85.325	85.164	85.104	85.124	85.215	85.366

Table 12
Specific enthalpy, $h(S, t, p)$, in kJ/kg, Eq. (13)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	0.061	0.119	0.155	0.168	0.158	0.127	0.074	0.000	−0.096
0	5	21.120	20.999	20.865	20.714	20.546	20.359	20.156	19.934	19.694
0	10	42.119	41.831	41.539	41.235	40.917	40.586	40.240	39.879	39.503
0	15	63.077	62.633	62.191	61.742	61.284	60.814	60.333	59.839	59.332
0	20	84.007	83.415	82.831	82.243	81.649	81.047	80.436	79.814	79.181
0	25	104.920	104.185	103.464	102.743	102.018	101.287	100.549	99.803	99.047
0	30	125.823	124.950	124.095	123.244	122.391	121.534	120.672	119.803	118.927
0	35	146.720	145.712	144.727	143.748	142.769	141.789	140.805	139.817	138.822
0	40	167.616	166.476	165.363	164.258	163.156	162.055	160.951	159.844	158.732
10	0	10.171	10.134	10.081	10.009	9.917	9.806	9.676	9.527	9.359
10	5	31.010	30.807	30.596	30.370	30.128	29.870	29.596	29.306	29.000
10	10	51.816	51.457	51.096	50.725	50.341	49.945	49.536	49.113	48.676
10	15	72.602	72.094	71.590	71.081	70.563	70.035	69.496	68.946	68.384
10	20	93.375	92.725	92.085	91.442	90.794	90.139	89.475	88.801	88.118
10	25	114.143	113.356	112.583	111.812	111.037	110.257	109.471	108.677	107.875
10	30	134.910	133.990	133.089	132.191	131.293	130.391	129.484	128.571	127.651
10	35	155.680	154.631	153.605	152.584	151.563	150.541	149.515	148.484	147.447
10	40	176.454	175.282	174.134	172.992	171.851	170.710	169.566	168.417	167.263
20	0	20.133	20.008	19.871	19.719	19.549	19.363	19.161	18.941	18.704
20	5	40.774	40.494	40.209	39.912	39.600	39.275	38.935	38.580	38.210
20	10	61.404	60.977	60.551	60.115	59.669	59.212	58.743	58.260	57.765
20	15	82.031	81.463	80.900	80.333	79.758	79.174	78.580	77.976	77.361
20	20	102.660	101.955	101.260	100.565	99.865	99.158	98.444	97.721	96.990
20	25	123.293	122.456	121.634	120.814	119.991	119.164	118.332	117.493	116.646
20	30	143.934	142.969	142.023	141.081	140.138	139.192	138.242	137.287	136.325
20	35	164.585	163.497	162.430	161.368	160.307	159.243	158.176	157.104	156.027
20	40	185.245	184.041	182.858	181.680	180.502	179.321	178.137	176.948	175.753
40	0	39.651	39.363	39.073	38.773	38.461	38.138	37.801	37.452	37.089
40	5	59.948	59.528	59.107	58.678	58.238	57.787	57.324	56.849	56.361
40	10	80.271	79.720	79.173	78.619	78.056	77.484	76.902	76.309	75.705
40	15	100.620	99.939	99.267	98.591	97.910	97.222	96.525	95.820	95.104
40	20	120.992	120.184	119.389	118.594	117.797	116.994	116.186	115.370	114.546
40	25	141.386	140.453	139.539	138.627	137.715	136.799	135.879	134.954	134.022
40	30	161.800	160.749	159.718	158.691	157.664	156.636	155.604	154.568	153.526
40	35	182.235	181.071	179.928	178.788	177.649	176.507	175.362	174.211	173.055
40	40	202.687	201.423	200.173	198.924	197.672	196.417	195.156	193.888	192.613
100	0	95.496	94.811	94.145	93.483	92.823	92.162	91.499	90.832	90.161
100	5	115.076	114.312	113.559	112.805	112.048	111.287	110.520	109.746	108.966
100	10	134.750	133.893	133.046	132.197	131.345	130.487	129.623	128.752	127.874
100	15	154.498	153.539	152.592	151.646	150.698	149.746	148.788	147.825	146.855
100	20	174.307	173.241	172.192	171.146	170.100	169.052	168.001	166.946	165.886
100	25	194.167	192.994	191.841	190.694	189.549	188.403	187.256	186.105	184.951
100	30	214.070	212.796	211.542	210.293	209.046	207.799	206.549	205.297	204.040
100	35	234.010	232.647	231.298	229.949	228.598	227.244	225.884	224.519	223.148
100	40	253.981	252.551	251.116	249.672	248.216	246.748	245.269	243.778	242.274

Table 13
Specific internal energy, $e(S, t, p)$, in kJ/kg, Eq. (14)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	−0.040	0.008	0.045	0.062	0.057	0.029	−0.022	−0.099	−0.201
0	5	21.019	20.892	20.760	20.612	20.446	20.262	20.059	19.836	19.591
0	10	42.018	41.728	41.437	41.134	40.818	40.487	40.141	39.779	39.398
0	15	62.975	62.532	62.092	61.643	61.185	60.716	60.235	59.741	59.230
0	20	83.906	83.315	82.733	82.146	81.552	80.950	80.339	79.718	79.083
0	25	104.819	104.086	103.366	102.644	101.919	101.188	100.451	99.705	98.948
0	30	125.721	124.850	123.995	123.142	122.288	121.432	120.571	119.704	118.825
0	35	146.618	145.612	144.626	143.645	142.666	141.686	140.704	139.717	138.720
0	40	167.514	166.375	165.261	164.154	163.051	161.951	160.850	159.745	158.631
10	0	0.118	0.116	0.100	0.069	0.021	−0.048	−0.138	−0.252	−0.391
10	5	20.957	20.791	20.618	20.432	20.230	20.013	19.777	19.522	19.246
10	10	41.759	41.437	41.114	40.781	40.436	40.078	39.705	39.317	38.912
10	15	62.538	62.067	61.602	61.130	60.649	60.158	59.656	59.142	58.613
10	20	83.301	82.689	82.088	81.483	80.871	80.252	79.625	78.988	78.338
10	25	104.057	103.309	102.575	101.840	101.101	100.358	99.608	98.850	98.080
10	30	124.810	123.929	123.065	122.203	121.340	120.474	119.604	118.727	117.837
10	35	145.562	144.551	143.560	142.573	141.588	140.603	139.616	138.622	137.615
10	40	166.318	165.177	164.058	162.947	161.842	160.741	159.640	158.532	157.410
20	0	0.227	0.179	0.114	0.038	−0.051	−0.158	−0.284	−0.432	−0.606
20	5	20.863	20.660	20.449	20.227	19.993	19.743	19.477	19.193	18.889
20	10	41.483	41.130	40.777	40.416	40.044	39.660	39.263	38.851	38.424
20	15	62.094	61.598	61.110	60.616	60.113	59.602	59.080	58.547	58.001
20	20	82.702	82.069	81.450	80.828	80.200	79.565	78.922	78.270	77.607
20	25	103.310	102.546	101.800	101.053	100.301	99.545	98.784	98.014	97.233
20	30	123.921	123.030	122.159	121.288	120.416	119.542	118.663	117.776	116.876
20	35	144.537	143.522	142.526	141.533	140.542	139.552	138.559	137.558	136.541
20	40	165.160	164.016	162.893	161.779	160.672	159.569	158.467	157.356	156.228
40	0	0.308	0.183	0.028	−0.129	−0.292	−0.466	−0.657	−0.867	−1.101
40	5	20.586	20.320	20.039	19.753	19.458	19.152	18.831	18.493	18.138
40	10	40.880	40.473	40.066	39.655	39.234	38.803	38.361	37.906	37.438
40	15	61.190	60.646	60.117	59.584	59.043	58.494	57.936	57.369	56.792
40	20	81.514	80.842	80.193	79.541	78.883	78.219	77.548	76.869	76.182
40	25	101.852	101.059	100.292	99.524	98.750	97.971	97.187	96.396	95.594
40	30	122.204	121.295	120.411	119.526	118.637	117.746	116.850	115.945	115.026
40	35	142.569	141.544	140.542	139.540	138.538	137.536	136.532	135.517	134.482
40	40	162.946	161.799	160.671	159.549	158.436	157.329	156.223	155.105	153.962
100	0	−0.266	−0.534	−0.905	−1.262	−1.602	−1.933	−2.268	−2.616	−2.986
100	5	19.206	18.815	18.370	17.930	17.493	17.055	16.610	16.153	15.682
100	10	38.756	38.231	37.702	37.173	36.642	36.105	35.562	35.011	34.452
100	15	58.366	57.712	57.093	56.474	55.848	55.215	54.577	53.936	53.291
100	20	78.025	77.252	76.542	75.830	75.108	74.377	73.642	72.904	72.164
100	25	97.722	96.847	96.045	95.235	94.412	93.580	92.741	91.898	91.049
100	30	117.450	116.486	115.588	114.677	113.749	112.811	111.866	110.913	109.945
100	35	137.205	136.152	135.151	134.132	133.099	132.057	131.009	129.948	128.862
100	40	156.979	155.825	154.703	153.568	152.430	151.293	150.155	149.005	147.820

Note that this Table 14 is in J/(kg K), not kJ/(kg K).

Table 14
Specific entropy, $\sigma(S, t, p)$, in kJ/(kg K), Eq. (3)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	−0.148	2.484	3.344	3.492	3.149	2.416	1.353	0.000	−1.616
0	5	76.252	78.234	78.478	78.032	77.111	75.817	74.207	72.318	70.178
0	10	151.077	152.466	152.145	151.153	149.702	147.890	145.773	143.388	140.762
0	15	224.448	225.290	224.447	222.947	221.001	218.706	216.115	213.265	210.183
0	20	296.463	296.794	295.461	293.485	291.073	288.321	285.282	281.992	278.476
0	25	367.200	367.049	365.252	362.823	359.968	356.781	353.314	349.602	345.670
0	30	436.725	436.116	433.875	431.013	427.732	424.126	420.247	416.129	411.796
0	35	505.097	504.046	501.378	498.098	494.407	490.397	486.119	481.608	476.886
0	40	572.365	570.888	567.808	564.124	560.036	555.634	550.971	546.078	540.981
10	0	0.341	2.774	3.453	3.432	2.929	2.046	0.840	−0.648	−2.392
10	5	75.943	77.774	77.878	77.298	76.251	74.835	73.108	71.108	68.862
10	10	150.081	151.354	150.925	149.828	148.277	146.368	144.159	141.685	138.973
10	15	222.848	223.601	222.673	221.093	219.069	216.698	214.035	211.116	207.967
10	20	294.323	294.585	293.187	291.149	288.678	285.868	282.775	279.432	275.866
10	25	364.570	364.368	362.523	360.048	357.149	353.919	350.411	346.661	342.693
10	30	433.645	433.003	430.729	427.835	424.523	420.887	416.979	412.833	408.473
10	35	501.598	500.536	497.852	494.554	490.844	486.813	482.515	477.982	473.239
10	40	568.474	567.013	563.937	560.250	556.154	551.741	547.061	542.150	537.031
20	0	0.469	2.722	3.237	3.062	2.414	1.394	0.059	−1.552	−3.412
20	5	75.350	77.044	77.020	76.319	75.156	73.630	71.797	69.696	67.353
20	10	148.862	150.030	149.502	148.311	146.668	144.671	142.377	139.822	137.031
20	15	221.075	221.747	220.743	219.088	216.994	214.554	211.826	208.844	205.634
20	20	292.051	292.252	290.795	288.701	286.175	283.314	280.171	276.781	273.170
20	25	361.844	361.597	359.708	357.191	354.252	350.984	347.440	343.656	339.655
20	30	430.500	429.828	427.525	424.603	421.263	417.600	413.666	409.496	405.113
20	35	498.062	496.990	494.293	490.979	487.252	483.203	478.886	474.334	469.572
20	40	564.570	563.125	560.054	556.365	552.262	547.837	543.143	538.214	533.074
40	0	−0.246	1.692	1.919	1.477	0.578	−0.679	−2.237	−4.058	−6.116
40	5	73.389	74.846	74.600	73.688	72.325	70.606	68.590	66.313	63.801
40	10	145.807	146.797	146.097	144.742	142.941	140.792	138.351	135.654	132.728
40	15	217.045	217.580	216.445	214.664	212.447	209.890	207.048	203.957	200.642
40	20	287.137	287.234	285.677	283.487	280.870	277.921	274.693	271.223	267.535
40	25	356.118	355.796	353.833	351.247	348.241	344.910	341.306	337.465	333.411
40	30	424.022	423.303	420.952	417.983	414.598	410.891	406.915	402.705	398.284
40	35	490.878	489.793	487.075	483.737	479.982	475.905	471.557	466.974	462.181
40	40	556.718	555.307	552.246	548.556	544.441	539.997	535.276	530.315	525.138
100	0	−8.535	−7.277	−7.661	−8.667	−10.088	−11.830	−13.839	−16.080	−18.528
100	5	62.499	63.470	62.771	61.430	59.658	57.551	55.166	52.538	49.694
100	10	132.600	133.242	132.208	130.529	128.417	125.967	123.237	120.262	117.069
100	15	201.735	202.020	200.638	198.618	196.170	193.389	190.331	187.033	183.519
100	20	269.891	269.807	268.072	265.708	262.925	259.816	256.437	252.822	248.998
100	25	337.067	336.620	334.535	331.829	328.709	325.269	321.563	317.627	313.485
100	30	403.269	402.487	400.064	397.020	393.561	389.782	385.737	381.461	376.979
100	35	468.508	467.436	464.701	461.330	457.532	453.402	448.998	444.353	439.494
100	40	532.797	531.507	528.499	524.818	520.682	516.189	511.399	506.348	501.065

Table 15
Chemical potential of water in seawater, $\mu^w(S, t, p)$, in kJ/kg, Eq. (15)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	0.101	−0.234	−0.562	−0.890	−1.222	−1.559	−1.903	−2.252	−2.605
0	5	−0.090	−0.431	−0.765	−1.101	−1.440	−1.785	−2.137	−2.495	−2.857
0	10	−0.659	−1.006	−1.347	−1.689	−2.035	−2.388	−2.748	−3.114	−3.485
0	15	−1.598	−1.952	−2.299	−2.647	−3.000	−3.360	−3.728	−4.102	−4.482
0	20	−2.901	−3.261	−3.614	−3.968	−4.328	−4.695	−5.070	−5.452	−5.840
0	25	−4.560	−4.926	−5.285	−5.646	−6.013	−6.386	−6.769	−7.158	−7.554
0	30	−6.571	−6.943	−7.307	−7.674	−8.047	−8.428	−8.817	−9.214	−9.618
0	35	−8.926	−9.303	−9.674	−10.047	−10.426	−10.813	−11.209	−11.614	−12.025
0	40	−11.620	−12.003	−12.380	−12.759	−13.144	−13.537	−13.940	−14.352	−14.770
10	0	10.078	9.742	9.414	9.085	8.752	8.413	8.068	7.717	7.361
10	5	9.886	9.545	9.210	8.874	8.533	8.187	7.834	7.474	7.110
10	10	9.321	8.973	8.632	8.289	7.942	7.588	7.227	6.859	6.486
10	15	8.388	8.034	7.686	7.337	6.983	6.622	6.254	5.878	5.497
10	20	7.094	6.734	6.381	6.025	5.665	5.297	4.921	4.537	4.148
10	25	5.447	5.081	4.721	4.360	3.992	3.618	3.235	2.844	2.446
10	30	3.451	3.079	2.713	2.346	1.972	1.591	1.201	0.802	0.397
10	35	1.112	0.734	0.363	−0.011	−0.390	−0.778	−1.175	−1.581	−1.993
10	40	−1.564	−1.947	−2.324	−2.704	−3.090	−3.484	−3.888	−4.301	−4.721
20	0	20.005	19.669	19.340	19.010	18.676	18.336	17.989	17.637	17.279
20	5	19.815	19.473	19.137	18.801	18.459	18.112	17.757	17.396	17.030
20	10	19.254	18.905	18.564	18.220	17.872	17.518	17.155	16.786	16.412
20	15	18.328	17.974	17.626	17.276	16.922	16.560	16.190	15.813	15.430
20	20	17.045	16.685	16.331	15.975	15.613	15.245	14.868	14.483	14.092
20	25	15.410	15.043	14.684	14.321	13.954	13.578	13.194	12.802	12.403
20	30	13.428	13.056	12.690	12.322	11.948	11.566	11.175	10.775	10.369
20	35	11.107	10.728	10.357	9.983	9.602	9.214	8.816	8.409	7.995
20	40	8.450	8.066	7.688	7.308	6.921	6.526	6.121	5.707	5.286
40	0	39.718	39.381	39.051	38.720	38.384	38.041	37.692	37.337	36.976
40	5	39.535	39.192	38.855	38.517	38.174	37.825	37.468	37.105	36.736
40	10	38.986	38.637	38.294	37.950	37.600	37.244	36.880	36.508	36.131
40	15	38.078	37.723	37.375	37.024	36.668	36.304	35.932	35.553	35.168
40	20	36.817	36.457	36.102	35.745	35.382	35.011	34.633	34.246	33.852
40	25	35.209	34.842	34.481	34.118	33.749	33.372	32.986	32.592	32.191
40	30	33.258	32.885	32.519	32.149	31.774	31.390	30.998	30.596	30.188
40	35	30.970	30.592	30.219	29.844	29.462	29.072	28.673	28.264	27.848
40	40	28.351	27.966	27.588	27.207	26.819	26.422	26.015	25.599	25.176
100	0	97.827	97.488	97.155	96.820	96.479	96.132	95.778	95.417	95.050
100	5	97.692	97.347	97.008	96.666	96.320	95.966	95.605	95.236	94.861
100	10	97.204	96.853	96.507	96.160	95.807	95.446	95.078	94.701	94.319
100	15	96.368	96.011	95.659	95.305	94.945	94.578	94.203	93.819	93.429
100	20	95.188	94.825	94.468	94.107	93.741	93.368	92.985	92.594	92.197
100	25	93.670	93.301	92.938	92.572	92.200	91.819	91.430	91.032	90.627
100	30	91.819	91.444	91.075	90.703	90.324	89.937	89.541	89.136	88.724
100	35	89.639	89.259	88.883	88.505	88.120	87.727	87.324	86.912	86.492
100	40	87.136	86.749	86.367	85.983	85.591	85.191	84.781	84.361	83.933

Table 16
Density anomaly, $\gamma(S, t, p)$, in kg/m^3, Eq. (2)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	−0.1569	3.9148	7.9557	11.9874	16.0161	20.0447	24.0749	28.1072	32.1419
0	5	−0.0334	3.9478	7.9038	11.8542	15.8046	19.7576	23.7144	27.6756	31.6414
0	10	−0.2975	3.6117	7.4989	11.3827	15.2684	19.1582	23.0533	26.9541	30.8608
0	15	−0.8974	2.9534	6.7844	10.6133	14.4452	18.2824	22.1258	25.9760	29.8331
0	20	−1.7929	2.0097	5.7939	9.5771	13.3643	17.1575	20.9580	24.7660	28.5816
0	25	−2.9524	0.8100	4.5551	8.3001	12.0498	15.8064	19.5709	23.3436	27.1248
0	30	−4.3505	−0.6207	3.0921	6.8053	10.5236	14.2493	17.9833	21.7261	25.4778
0	35	−5.9667	−2.2610	1.4267	5.1142	8.8064	12.5058	16.2134	19.9298	23.6550
0	40	−7.7836	−4.0910	−0.4202	3.2481	6.9193	10.5962	14.2800	17.9715	21.6708
10	0	4.8718	8.8944	12.8883	16.8743	20.8584	24.8433	28.8304	32.8204	36.8135
10	5	4.8289	8.7671	12.6818	16.5919	20.5029	24.4172	28.3359	32.2596	36.1883
10	10	4.4305	8.3016	12.1520	15.9998	19.8501	23.7051	27.5658	31.4328	35.3060
10	15	3.7224	7.5390	11.3366	15.1329	18.9326	22.7380	26.5500	30.3692	34.1955
10	20	2.7401	6.5115	10.2652	14.0184	17.7759	21.5399	25.3113	29.0906	32.8778
10	25	1.5117	5.2454	8.9622	12.6793	16.4015	20.1308	23.8682	27.6143	31.3689
10	30	0.0601	3.7629	7.4492	11.1362	14.8287	18.5288	22.2374	25.9552	29.6821
10	35	−1.5961	2.0838	5.7461	9.4086	13.0763	16.7513	20.4349	24.1276	27.8294
10	40	−3.4413	0.2259	3.8719	7.5158	11.1631	14.8164	18.4770	22.1456	25.8224
20	0	9.7851	13.7616	17.7107	21.6529	25.5940	29.5369	33.4828	37.4324	41.3859
20	5	9.5844	13.4821	17.3572	21.2285	25.1014	28.9783	32.8603	36.7480	40.6414
20	10	9.0578	12.8930	16.7080	20.5209	24.3369	28.1581	31.9856	35.8200	39.6612
20	15	8.2457	12.0300	15.7956	19.5601	23.3286	27.1031	30.8848	34.6741	38.4710
20	20	7.1793	10.9214	14.6458	18.3699	22.0986	25.8342	29.5776	33.3293	37.0893
20	25	5.8839	9.5905	13.2803	16.9704	20.6659	24.3688	28.0803	31.8006	35.5300
20	30	4.3797	8.0574	11.7184	15.3801	19.0476	22.7229	26.4073	30.1010	33.8043
20	35	2.6837	6.3397	9.9779	13.6164	17.2603	20.9120	24.5726	28.2426	31.9221
20	40	0.8099	4.4538	8.0763	11.6969	15.3212	18.9519	22.5903	26.2371	29.8926
40	0	19.2789	23.1712	27.0364	30.8960	34.7560	38.6194	42.4877	46.3616	50.2414
40	5	18.7879	22.6113	26.4117	30.2092	34.0094	37.8150	41.6274	45.4472	49.2746
40	10	18.0229	21.7919	25.5398	29.2861	33.0363	36.7929	40.5573	44.3301	48.1115
40	15	17.0151	20.7401	24.4449	28.1488	31.8572	35.5727	39.2967	43.0296	46.7718
40	20	15.7892	19.4777	23.1465	26.8150	30.4885	34.1697	37.8598	41.5596	45.2691
40	25	14.3652	18.0229	21.6615	25.3001	28.9443	32.5968	36.2588	39.9309	43.6135
40	30	12.7594	16.3917	20.0049	23.6184	27.2380	30.8661	34.5041	38.1528	41.8124
40	35	10.9852	14.5984	18.1913	21.7841	25.3825	28.9893	32.6060	36.2333	39.8716
40	40	9.0540	12.6565	16.2351	19.8113	23.3915	26.9788	30.5747	34.1803	37.7960
100	0	45.3191	49.0158	52.6680	56.3118	59.9584	63.6139	67.2821	70.9654	74.6655
100	5	44.1398	47.7878	51.3949	54.9959	58.6019	62.2188	65.8501	69.4982	73.1648
100	10	42.7940	46.4060	49.9783	53.5454	57.1184	60.7031	64.3030	67.9206	71.5575
100	15	41.2987	44.8831	48.4277	51.9674	55.5133	59.0712	62.6448	66.2363	69.8476
100	20	39.6671	43.2290	46.7508	50.2676	53.7907	57.3260	60.8773	64.4468	68.0364
100	25	37.9095	41.4521	44.9542	48.4513	51.9547	55.4705	59.0024	62.5529	66.1237
100	30	36.0337	39.5601	43.0452	46.5247	50.0105	53.5085	57.0226	60.5554	64.1087
100	35	34.0463	37.5605	41.0311	44.4950	47.9641	51.4449	54.9412	58.4558	61.9904
100	40	31.9524	35.4608	38.9206	42.3708	45.8238	49.2866	52.7632	56.2564	59.7684

Table 17
Isobaric specific heat capacity, $c_P(S, t, p)$, in J/kgK, Eq. (7)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	4219.41	4182.14	4146.85	4112.85	4079.90	4047.84	4016.60	3986.08	3956.25
0	5	4205.05	4170.59	4137.85	4106.23	4075.51	4045.58	4016.34	3987.74	3959.73
0	10	4195.14	4162.96	4132.26	4102.53	4073.58	4045.30	4017.62	3990.49	3963.87
0	15	4188.45	4158.07	4128.96	4100.68	4073.08	4046.05	4019.55	3993.52	3967.93
0	20	4184.06	4155.05	4127.14	4099.96	4073.36	4047.26	4021.62	3996.39	3971.53
0	25	4181.32	4153.33	4126.31	4099.93	4074.07	4048.64	4023.62	3998.96	3974.63
0	30	4179.81	4152.56	4126.20	4100.41	4075.09	4050.17	4025.61	4001.37	3977.44
0	35	4179.25	4152.55	4126.69	4101.37	4076.49	4051.98	4027.82	4003.96	3980.39
0	40	4179.42	4153.18	4127.75	4102.87	4078.42	4054.34	4030.59	4007.16	3984.00
10	0	4172.22	4137.92	4105.12	4073.31	4042.29	4011.96	3982.25	3953.09	3924.46
10	5	4164.06	4131.87	4101.11	4071.29	4042.21	4013.78	3985.94	3958.62	3931.80
10	10	4158.81	4128.35	4099.21	4070.94	4043.37	4016.40	3989.97	3964.02	3938.54
10	15	4155.65	4126.59	4098.73	4071.65	4045.20	4019.30	3993.89	3968.93	3944.37
10	20	4153.98	4126.06	4099.19	4073.01	4047.39	4022.25	3997.54	3973.22	3949.26
10	25	4153.37	4126.41	4100.34	4074.85	4049.82	4025.20	4000.94	3977.01	3953.39
10	30	4153.55	4127.45	4102.06	4077.12	4052.55	4028.29	4004.32	3980.61	3957.14
10	35	4154.32	4129.09	4104.35	4079.92	4055.75	4031.79	4008.04	3984.46	3961.05
10	40	4155.54	4131.28	4107.26	4083.39	4059.66	4036.04	4012.52	3989.08	3965.74
20	0	4129.67	4098.10	4067.58	4037.74	4008.46	3979.65	3951.27	3923.28	3895.65
20	5	4126.81	4096.71	4067.77	4039.57	4011.97	3984.91	3958.30	3932.13	3906.35
20	10	4125.56	4096.68	4068.97	4042.04	4015.72	3989.95	3964.65	3939.79	3915.34
20	15	4125.46	4097.60	4070.88	4044.90	4019.52	3994.66	3970.26	3946.29	3922.70
20	20	4126.15	4099.22	4073.30	4048.05	4023.34	3999.08	3975.24	3951.78	3928.66
20	25	4127.40	4101.38	4076.18	4051.50	4027.25	4003.37	3979.83	3956.58	3933.62
20	30	4129.06	4104.02	4079.51	4055.35	4031.46	4007.81	3984.38	3961.14	3938.08
20	35	4130.99	4107.12	4083.40	4059.78	4036.24	4012.76	3989.34	3965.98	3942.67
20	40	4133.13	4110.71	4087.97	4065.02	4041.90	4018.63	3995.23	3971.72	3948.09
40	0	4056.52	4029.87	4003.38	3976.99	3950.68	3924.45	3898.28	3872.16	3846.10
40	5	4062.15	4035.78	4010.03	3984.69	3959.68	3934.96	3910.48	3886.24	3862.21
40	10	4067.29	4041.19	4016.02	3991.44	3967.35	3943.68	3920.38	3897.42	3874.78
40	15	4072.09	4046.35	4021.64	3997.62	3974.14	3951.13	3928.54	3906.34	3884.50
40	20	4076.61	4051.40	4027.17	4003.56	3980.46	3957.80	3935.53	3913.62	3892.04
40	25	4080.89	4056.50	4032.81	4009.58	3986.71	3964.15	3941.88	3919.86	3898.08
40	30	4084.93	4061.76	4038.81	4015.99	3993.28	3970.67	3948.15	3925.70	3903.33
40	35	4088.75	4067.30	4045.37	4023.12	4000.61	3977.88	3954.95	3931.83	3908.55
40	40	4092.36	4073.28	4052.77	4031.34	4009.16	3986.33	3962.92	3938.99	3914.57
100	0	3905.04	3890.81	3874.14	3855.87	3836.32	3815.65	3793.99	3771.43	3748.02
100	5	3926.12	3908.86	3890.72	3872.01	3852.85	3833.28	3813.35	3793.10	3772.56
100	10	3942.68	3923.08	3903.71	3884.48	3865.37	3846.36	3827.44	3808.60	3789.84
100	15	3956.09	3934.99	3914.71	3894.97	3875.67	3856.74	3838.14	3819.85	3801.84
100	20	3967.22	3945.60	3924.87	3904.72	3885.05	3865.78	3846.88	3828.31	3810.05
100	25	3976.60	3955.56	3934.94	3914.60	3894.47	3874.53	3854.76	3835.14	3815.67
100	30	3984.51	3965.34	3945.52	3925.29	3904.71	3883.84	3862.70	3841.33	3819.73
100	35	3991.22	3975.35	3957.16	3937.46	3916.53	3894.55	3871.63	3847.85	3823.27
100	40	3997.14	3986.17	3970.57	3951.95	3930.89	3907.72	3882.68	3855.91	3827.56

Table 18
Sound speed, $U(S, t, p)$, in m/s, Eq. (9)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	1402.40	1409.23	1415.88	1422.48	1429.08	1435.70	1442.34	1449.02	1455.76
0	5	1426.18	1432.71	1439.07	1445.38	1451.67	1457.97	1464.29	1470.65	1477.04
0	10	1447.28	1453.50	1459.58	1465.61	1471.63	1477.66	1483.71	1489.79	1495.91
0	15	1465.94	1471.87	1477.69	1483.47	1489.24	1495.03	1500.83	1506.67	1512.55
0	20	1482.35	1488.03	1493.61	1499.17	1504.72	1510.28	1515.87	1521.48	1527.13
0	25	1496.71	1502.17	1507.54	1512.89	1518.24	1523.59	1528.96	1534.36	1539.78
0	30	1509.16	1514.43	1519.61	1524.78	1529.94	1535.10	1540.28	1545.47	1550.70
0	35	1519.85	1524.93	1529.95	1534.95	1539.96	1544.97	1550.00	1555.05	1560.12
0	40	1528.91	1533.80	1538.67	1543.56	1548.46	1553.39	1558.36	1563.37	1568.42
10	0	1418.50	1425.27	1431.94	1438.60	1445.25	1451.90	1458.57	1465.24	1471.93
10	5	1442.33	1448.79	1455.17	1461.53	1467.88	1474.23	1480.58	1486.94	1493.30
10	10	1463.53	1469.67	1475.76	1481.83	1487.91	1493.98	1500.06	1506.14	1512.22
10	15	1482.32	1488.15	1493.97	1499.78	1505.59	1511.41	1517.24	1523.06	1528.89
10	20	1498.90	1504.45	1510.02	1515.58	1521.16	1526.73	1532.31	1537.89	1543.47
10	25	1513.44	1518.75	1524.08	1529.42	1534.76	1540.11	1545.46	1550.80	1556.13
10	30	1526.09	1531.17	1536.29	1541.42	1546.56	1551.70	1556.83	1561.96	1567.07
10	35	1536.99	1541.85	1546.77	1551.72	1556.69	1561.66	1566.63	1571.60	1576.56
10	40	1546.27	1550.90	1555.65	1560.46	1565.31	1570.19	1575.10	1580.03	1584.97
20	0	1434.92	1441.62	1448.33	1455.03	1461.74	1468.45	1475.14	1481.81	1488.45
20	5	1458.71	1465.09	1471.50	1477.91	1484.32	1490.72	1497.10	1503.46	1509.79
20	10	1479.93	1485.98	1492.08	1498.20	1504.33	1510.45	1516.55	1522.63	1528.67
20	15	1498.79	1504.51	1510.33	1516.17	1522.03	1527.88	1533.72	1539.52	1545.29
20	20	1515.47	1520.90	1526.44	1532.03	1537.63	1543.22	1548.80	1554.34	1559.84
20	25	1530.14	1535.30	1540.60	1545.94	1551.29	1556.64	1561.97	1567.26	1572.50
20	30	1542.95	1547.85	1552.92	1558.04	1563.17	1568.30	1573.40	1578.47	1583.48
20	35	1554.02	1558.68	1563.53	1568.45	1573.41	1578.36	1583.31	1588.22	1593.09
20	40	1563.49	1567.88	1572.54	1577.32	1582.16	1587.03	1591.92	1596.81	1601.67
40	0	1468.66	1475.21	1481.97	1488.80	1495.64	1502.46	1509.21	1515.88	1522.44
40	5	1492.11	1498.32	1504.77	1511.30	1517.85	1524.35	1530.80	1537.15	1543.39
40	10	1513.17	1519.01	1525.15	1531.37	1537.61	1543.82	1549.96	1556.00	1561.92
40	15	1531.99	1537.48	1543.30	1549.22	1555.17	1561.08	1566.92	1572.66	1578.27
40	20	1548.72	1553.89	1559.41	1565.05	1570.71	1576.34	1581.89	1587.34	1592.65
40	25	1563.51	1568.37	1573.62	1579.00	1584.40	1589.78	1595.07	1600.25	1605.29
40	30	1576.48	1581.06	1586.07	1591.21	1596.40	1601.56	1606.64	1611.61	1616.44
40	35	1587.77	1592.07	1596.85	1601.81	1606.83	1611.85	1616.81	1621.67	1626.41
40	40	1597.50	1601.49	1606.08	1610.91	1615.85	1620.82	1625.79	1630.70	1635.52
100	0	1575.62	1581.32	1588.09	1595.16	1602.26	1609.22	1615.89	1622.18	1627.98
100	5	1596.65	1602.03	1608.50	1615.26	1622.04	1628.65	1634.96	1640.84	1646.22
100	10	1615.89	1620.90	1627.03	1633.46	1639.89	1646.13	1652.04	1657.51	1662.43
100	15	1633.37	1638.01	1643.81	1649.92	1656.03	1661.93	1667.48	1672.57	1677.10
100	20	1649.12	1653.44	1658.97	1664.82	1670.66	1676.30	1681.58	1686.39	1690.62
100	25	1663.24	1667.27	1672.58	1678.24	1683.92	1689.40	1694.52	1699.18	1703.26
100	30	1675.80	1679.55	1684.69	1690.24	1695.84	1701.28	1706.40	1711.07	1715.18
100	35	1686.89	1690.33	1695.30	1700.78	1706.39	1711.91	1717.17	1722.03	1726.39
100	40	1696.61	1699.58	1704.33	1709.74	1715.42	1721.10	1726.64	1731.88	1736.72

Table 19
Potential temperature, $\theta(S,t,p,0)$, in °C, Eq. (16)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	5	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
0	10	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
0	15	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
0	20	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000
0	25	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000
0	30	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000
0	35	35.0000	35.0000	35.0000	35.0000	35.0000	35.0000	35.0000	35.0000	35.0000
0	40	40.0000	40.0000	40.0000	40.0000	40.0000	40.0000	40.0000	40.0000	40.0000
10	0	0.0316	0.0190	0.0072	−0.0040	−0.0147	−0.0250	−0.0349	−0.0444	−0.0536
10	5	4.9796	4.9693	4.9596	4.9503	4.9413	4.9325	4.9239	4.9156	4.9076
10	10	9.9328	9.9244	9.9164	9.9086	9.9010	9.8935	9.8863	9.8792	9.8722
10	15	14.8900	14.8830	14.8763	14.8697	14.8633	14.8571	14.8509	14.8450	14.8391
10	20	19.8501	19.8442	19.8386	19.8330	19.8277	19.8224	19.8173	19.8123	19.8074
10	25	24.8125	24.8076	24.8029	24.7982	24.7937	24.7893	24.7850	24.7808	24.7767
10	30	29.7767	29.7728	29.7690	29.7651	29.7614	29.7576	29.7540	29.7504	29.7468
10	35	34.7422	34.7396	34.7368	34.7338	34.7308	34.7276	34.7244	34.7211	34.7177
10	40	39.7086	39.7079	39.7065	39.7045	39.7021	39.6994	39.6964	39.6931	39.6896
20	0	0.0399	0.0156	−0.0071	−0.0286	−0.0492	−0.0690	−0.0880	−0.1063	−0.1240
20	5	4.9403	4.9207	4.9020	4.8840	4.8666	4.8497	4.8332	4.8172	4.8016
20	10	9.8505	9.8344	9.8189	9.8039	9.7892	9.7748	9.7608	9.7471	9.7336
20	15	14.7680	14.7546	14.7416	14.7290	14.7166	14.7045	14.6927	14.6811	14.6698
20	20	19.6911	19.6797	19.6688	19.6581	19.6477	19.6376	19.6277	19.6180	19.6086
20	25	24.6183	24.6089	24.5997	24.5907	24.5819	24.5734	24.5651	24.5570	24.5491
20	30	29.5488	29.5413	29.5339	29.5265	29.5191	29.5119	29.5048	29.4978	29.4910
20	35	34.4817	34.4768	34.4714	34.4656	34.4596	34.4534	34.4471	34.4407	34.4342
20	40	39.4165	39.4152	39.4123	39.4083	39.4036	39.3983	39.3924	39.3860	39.3791
40	0	−0.0064	−0.0517	−0.0938	−0.1338	−0.1721	−0.2088	−0.2440	−0.2779	−0.3106
40	5	4.8107	4.7741	4.7394	4.7059	4.6735	4.6420	4.6113	4.5815	4.5524
40	10	9.6446	9.6147	9.5859	9.5579	9.5305	9.5037	9.4774	9.4518	9.4266
40	15	14.4912	14.4663	14.4421	14.4185	14.3955	14.3729	14.3508	14.3291	14.3079
40	20	19.3474	19.3263	19.3059	19.2860	19.2666	19.2476	19.2291	19.2111	19.1935
40	25	24.2109	24.1933	24.1761	24.1593	24.1430	24.1270	24.1115	24.0964	24.0817
40	30	29.0800	29.0661	29.0521	29.0382	29.0245	29.0109	28.9977	28.9846	28.9718
40	35	33.9534	33.9441	33.9338	33.9228	33.9115	33.8998	33.8879	33.8758	33.8635
40	40	38.8298	38.8273	38.8216	38.8140	38.8048	38.7944	38.7829	38.7704	38.7570
100	0	−0.5423	−0.6366	−0.7238	−0.8062	−0.8846	−0.9596	−1.0312	−1.0997	−1.1653
100	5	4.0920	4.0173	3.9463	3.8778	3.8115	3.7470	3.6845	3.6237	3.5646
100	10	8.7560	8.6958	8.6374	8.5803	8.5244	8.4697	8.4161	8.3636	8.3123
100	15	13.4420	13.3922	13.3434	13.2956	13.2486	13.2026	13.1574	13.1131	13.0698
100	20	18.1445	18.1024	18.0611	18.0207	17.9812	17.9425	17.9047	17.8678	17.8318
100	25	22.8594	22.8238	22.7888	22.7545	22.7210	22.6882	22.6563	22.6252	22.5950
100	30	27.5834	27.5550	27.5260	27.4971	27.4684	27.4400	27.4120	27.3844	27.3574
100	35	32.3140	32.2952	32.2732	32.2496	32.2248	32.1990	32.1726	32.1456	32.1181
100	40	37.0492	37.0445	37.0317	37.0140	36.9924	36.9675	36.9399	36.9097	36.8772

Table 20
Potential density anomaly, $\gamma_\theta(S,t,p,0)$, in kg/m^3, Eq. (17)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	−0.1569	3.9148	7.9557	11.9874	16.0161	20.0447	24.0749	28.1072	32.1419
0	5	−0.0334	3.9478	7.9038	11.8542	15.8046	19.7576	23.7144	27.6756	31.6414
0	10	−0.2975	3.6117	7.4989	11.3827	15.2684	19.1582	23.0533	26.9541	30.8608
0	15	−0.8974	2.9534	6.7844	10.6133	14.4452	18.2824	22.1258	25.9760	29.8331
0	20	−1.7929	2.0097	5.7939	9.5771	13.3643	17.1575	20.9580	24.7660	28.5816
0	25	−2.9524	0.8100	4.5551	8.3001	12.0498	15.8064	19.5709	23.3436	27.1248
0	30	−4.3505	−0.6207	3.0921	6.8053	10.5236	14.2493	17.9833	21.7261	25.4778
0	35	−5.9667	−2.2610	1.4267	5.1142	8.8064	12.5058	16.2134	19.9298	23.6550
0	40	−7.7836	−4.0910	−0.4202	3.2481	6.9193	10.5962	14.2800	17.9715	21.6708
10	0	−0.1548	3.9157	7.9559	11.9873	16.0162	20.0453	24.0762	28.1096	32.1456
10	5	−0.0330	3.9488	7.9057	11.8573	15.8091	19.7636	23.7222	27.6854	31.6533
10	10	−0.2916	3.6192	7.5083	11.3942	15.2819	19.1739	23.0714	26.9747	30.8840
10	15	−0.8809	2.9722	6.8055	10.6368	14.4712	18.3110	22.1571	26.0100	29.8697
10	20	−1.7620	2.0431	5.8299	9.6157	13.4055	17.2015	21.0046	24.8154	28.6338
10	25	−2.9044	0.8606	4.6084	8.3560	12.1084	15.8676	19.6348	23.4102	27.1940
10	30	−4.2833	−0.5510	3.1643	6.8801	10.6009	14.3292	18.0658	21.8112	25.5655
10	35	−5.8783	−2.1707	1.5191	5.2088	8.9034	12.6053	16.3154	20.0344	23.7624
10	40	−7.6725	−3.9792	−0.3071	3.3628	7.0361	10.7153	14.4017	18.0960	21.7984
20	0	−0.1542	3.9156	7.9555	11.9871	16.0164	20.0463	24.0783	28.1129	32.1504
20	5	−0.0324	3.9503	7.9084	11.8613	15.8147	19.7710	23.7315	27.6967	31.6668
20	10	−0.2845	3.6282	7.5192	11.4071	15.2970	19.1913	23.0912	26.9970	30.9089
20	15	−0.8628	2.9926	6.8283	10.6621	14.4990	18.3413	22.1900	26.0456	29.9080
20	20	−1.7296	2.0781	5.8674	9.6559	13.4483	17.2469	21.0528	24.8662	28.6872
20	25	−2.8551	0.9125	4.6629	8.4131	12.1681	15.9300	19.6997	23.4778	27.2642
20	30	−4.2151	−0.4805	3.2374	6.9556	10.6790	14.4098	18.1489	21.8969	25.6537
20	35	−5.7896	−2.0801	1.6118	5.3037	9.0006	12.7049	16.4175	20.1391	23.8697
20	40	−7.5617	−3.8677	−0.1943	3.4773	7.1526	10.8341	14.5231	18.2201	21.9255
40	0	−0.1574	3.9124	7.9530	11.9858	16.0170	20.0493	24.0840	28.1218	32.1628
40	5	−0.0306	3.9547	7.9157	11.8718	15.8288	19.7890	23.7537	27.7234	31.6981
40	10	−0.2671	3.6496	7.5447	11.4370	15.3315	19.2305	23.1353	27.0463	30.9634
40	15	−0.8222	3.0379	6.8783	10.7170	14.5589	18.4064	22.2603	26.1212	29.9891
40	20	−1.6604	2.1524	5.9468	9.7404	13.5380	17.3418	21.1528	24.9715	28.7978
40	25	−2.7529	1.0198	4.7753	8.5307	12.2907	16.0577	19.8325	23.6156	27.4070
40	30	−4.0764	−0.3370	3.3857	7.1087	10.8370	14.5727	18.3167	22.0696	25.8313
40	35	−5.6113	−1.8982	1.7978	5.4940	9.1955	12.9044	16.6219	20.3484	24.0840
40	40	−7.3412	−3.6457	0.0304	3.7054	7.3847	11.0707	14.7647	18.4671	22.1784
100	0	−0.1963	3.8812	7.9308	11.9738	16.0161	20.0603	24.1078	28.1589	32.2137
100	5	−0.0252	3.9722	7.9458	11.9152	15.8859	19.8605	23.8401	27.8250	31.8155
100	10	−0.1986	3.7321	7.6416	11.5485	15.4580	19.3724	23.2929	27.2199	31.1533
100	15	−0.6771	3.1978	7.0532	10.9070	14.7642	18.6272	22.4968	26.3736	30.2575
100	20	−1.4282	2.3995	6.2088	10.0174	13.8301	17.6490	21.4752	25.3090	29.1505
100	25	−2.4254	1.3620	5.1320	8.9019	12.6765	16.4579	20.2471	24.0446	27.8502
100	30	−3.6464	0.1064	3.8426	7.5794	11.3215	15.0710	18.8289	22.5956	26.3711
100	35	−5.0725	−1.3489	2.3588	6.0674	9.7819	13.5043	17.2356	20.9762	24.7263
100	40	−6.6879	−2.9874	0.6970	4.3824	8.0736	11.7730	15.4817	19.2000	22.9282

Table 21
Potential enthalpy, $h_\theta(S,t,p,0)$, in kJ/kg, Eq. (18)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	0.061	0.119	0.155	0.168	0.158	0.127	0.074	0.000	−0.096
0	5	21.120	20.999	20.865	20.714	20.546	20.359	20.156	19.934	19.694
0	10	42.119	41.831	41.539	41.235	40.917	40.586	40.240	39.879	39.503
0	15	63.077	62.633	62.191	61.742	61.284	60.814	60.333	59.839	59.332
0	20	84.007	83.415	82.831	82.243	81.649	81.047	80.436	79.814	79.181
0	25	104.920	104.185	103.464	102.743	102.018	101.287	100.549	99.803	99.047
0	30	125.823	124.950	124.095	123.244	122.391	121.534	120.672	119.803	118.927
0	35	146.720	145.712	144.727	143.748	142.769	141.789	140.805	139.817	138.822
0	40	167.616	166.476	165.363	164.258	163.156	162.055	160.951	159.844	158.732
10	0	0.194	0.198	0.185	0.151	0.098	0.026	−0.066	−0.177	−0.308
10	5	21.034	20.871	20.698	20.510	20.306	20.086	19.850	19.597	19.328
10	10	41.837	41.516	41.193	40.860	40.514	40.155	39.783	39.397	38.996
10	15	62.616	62.146	61.680	61.208	60.727	60.236	59.734	59.220	58.694
10	20	83.380	82.768	82.165	81.559	80.947	80.329	79.701	79.064	78.416
10	25	104.136	103.386	102.651	101.916	101.177	100.434	99.684	98.926	98.159
10	30	124.889	124.006	123.142	122.281	121.418	120.552	119.682	118.805	117.920
10	35	145.642	144.631	143.641	142.656	141.672	140.685	139.695	138.700	137.698
10	40	166.398	165.263	164.152	163.046	161.942	160.836	159.727	158.614	157.496
20	0	0.230	0.184	0.125	0.050	−0.042	−0.152	−0.279	−0.424	−0.586
20	5	20.869	20.668	20.459	20.238	20.002	19.751	19.486	19.205	18.908
20	10	41.492	41.142	40.791	40.430	40.059	39.675	39.279	38.870	38.447
20	15	62.105	61.612	61.124	60.631	60.129	59.619	59.098	58.566	58.022
20	20	82.715	82.084	81.464	80.842	80.214	79.580	78.938	78.287	77.627
20	25	103.324	102.561	101.812	101.065	100.315	99.560	98.799	98.031	97.255
20	30	123.937	123.045	122.172	121.302	120.431	119.557	118.679	117.794	116.903
20	35	144.554	143.540	142.546	141.556	140.566	139.574	138.579	137.577	136.570
20	40	165.178	164.048	162.937	161.831	160.724	159.615	158.502	157.384	156.259
40	0	0.034	−0.098	−0.234	−0.383	−0.544	−0.718	−0.906	−1.108	−1.324
40	5	20.324	20.057	19.787	19.506	19.215	18.911	18.595	18.265	17.922
40	10	40.628	40.227	39.828	39.421	39.005	38.578	38.141	37.692	37.230
40	15	60.945	60.413	59.888	59.358	58.821	58.277	57.723	57.160	56.586
40	20	81.277	80.616	79.966	79.316	78.662	78.002	77.336	76.661	75.978
40	25	101.621	100.835	100.064	99.296	98.526	97.753	96.974	96.189	95.397
40	30	121.977	121.071	120.184	119.300	118.415	117.528	116.637	115.741	114.838
40	35	142.346	141.328	140.327	139.330	138.332	137.331	136.326	135.316	134.298
40	40	162.726	161.606	160.499	159.392	158.282	157.167	156.046	154.917	153.781
100	0	−2.228	−2.544	−2.847	−3.149	−3.452	−3.757	−4.068	−4.384	−4.705
100	5	17.301	16.899	16.504	16.105	15.701	15.290	14.872	14.446	14.011
100	10	36.899	36.401	35.907	35.410	34.906	34.396	33.877	33.350	32.814
100	15	56.550	55.946	55.351	54.753	54.150	53.542	52.927	52.305	51.675
100	20	76.242	75.529	74.828	74.128	73.426	72.721	72.010	71.294	70.572
100	25	95.968	95.146	94.340	93.537	92.733	91.928	91.120	90.307	89.490
100	30	115.721	114.796	113.887	112.981	112.075	111.167	110.255	109.339	108.418
100	35	135.494	134.481	133.475	132.468	131.457	130.441	129.419	128.390	127.353
100	40	155.284	154.202	153.112	152.008	150.892	149.762	148.620	147.464	146.295

Table 22
Thermal expansion coefficient, $\alpha(S, t, p)$, in ppm/K, Eq. (6)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	−67.74	−47.17	−28.23	−10.37	6.59	22.76	38.21	52.99	67.13
0	5	16.03	31.98	46.95	61.27	75.03	88.29	101.09	113.47	125.43
0	10	87.94	100.45	112.36	123.85	134.97	145.76	156.23	166.41	176.30
0	15	150.84	160.80	170.35	179.59	188.56	197.27	205.74	213.97	221.97
0	20	206.80	214.80	222.48	229.91	237.12	244.11	250.89	257.46	263.83
0	25	257.29	263.60	269.70	275.63	281.38	286.95	292.36	297.60	302.66
0	30	303.38	307.98	312.58	317.15	321.65	326.07	330.41	334.66	338.81
0	35	345.89	348.44	351.41	354.62	357.99	361.47	365.04	368.68	372.38
0	40	385.48	385.32	386.32	388.03	390.28	392.99	396.08	399.52	403.27
10	0	−30.53	−11.59	5.85	22.27	37.87	52.71	66.89	80.43	93.37
10	5	45.64	60.31	74.12	87.32	100.02	112.27	124.10	135.54	146.59
10	10	111.49	122.99	133.99	144.61	154.91	164.91	174.63	184.08	193.27
10	15	169.45	178.62	187.45	196.00	204.32	212.40	220.27	227.93	235.37
10	20	221.29	228.67	235.78	242.67	249.35	255.83	262.12	268.22	274.13
10	25	268.27	274.11	279.77	285.26	290.58	295.75	300.75	305.58	310.25
10	30	311.33	315.59	319.87	324.10	328.28	332.37	336.38	340.30	344.13
10	35	351.17	353.54	356.31	359.31	362.46	365.72	369.05	372.45	375.89
10	40	388.39	388.23	389.20	390.85	393.03	395.64	398.62	401.94	405.55
20	0	4.24	21.72	37.80	52.94	67.29	80.95	93.96	106.38	118.23
20	5	73.54	87.04	99.76	111.95	123.68	135.00	145.93	156.49	166.71
20	10	133.84	144.40	154.54	164.35	173.87	183.13	192.14	200.91	209.44
20	15	187.24	195.65	203.78	211.68	219.38	226.86	234.16	241.26	248.16
20	20	235.24	242.01	248.56	254.92	261.09	267.08	272.89	278.52	283.98
20	25	278.93	284.30	289.51	294.57	299.48	304.23	308.83	313.28	317.56
20	30	319.13	323.05	326.99	330.89	334.74	338.51	342.20	345.79	349.29
20	35	356.45	358.61	361.18	363.96	366.88	369.91	373.01	376.17	379.37
20	40	391.42	391.23	392.14	393.71	395.80	398.31	401.18	404.37	407.85
40	0	66.96	81.93	95.68	108.59	120.79	132.37	143.38	153.84	163.78
40	5	124.44	135.88	146.70	157.07	167.07	176.71	186.03	195.03	203.73
40	10	175.05	183.91	192.48	200.81	208.91	216.81	224.51	232.01	239.31
40	15	220.35	227.36	234.20	240.88	247.40	253.76	259.97	266.01	271.91
40	20	261.45	267.08	272.56	277.91	283.11	288.16	293.06	297.82	302.42
40	25	299.18	303.63	307.98	312.22	316.33	320.31	324.16	327.87	331.44
40	30	334.15	337.38	340.67	343.94	347.16	350.33	353.42	356.42	359.34
40	35	366.85	368.58	370.72	373.07	375.58	378.18	380.87	383.61	386.39
40	40	397.69	397.35	398.11	399.51	401.43	403.75	406.43	409.43	412.71
100	0	208.29	218.42	227.49	235.81	243.50	250.61	257.20	263.29	268.90
100	5	242.47	249.54	256.18	262.48	268.48	274.21	279.66	284.86	289.80
100	10	272.93	277.94	282.85	287.64	292.31	296.84	301.25	305.51	309.64
100	15	300.65	304.34	308.07	311.76	315.39	318.95	322.44	325.83	329.12
100	20	326.28	329.13	332.05	334.96	337.83	340.65	343.39	346.06	348.65
100	25	350.30	352.52	354.85	357.20	359.53	361.83	364.08	366.27	368.38
100	30	373.07	374.59	376.38	378.30	380.28	382.30	384.34	386.38	388.40
100	35	394.84	395.35	396.49	398.01	399.79	401.78	403.95	406.26	408.69
100	40	415.81	414.71	414.94	415.99	417.67	419.89	422.57	425.66	429.12

Table 23
Isothermal compressibility $K(S, t, p)$ in ppm/MPa, Eq. (5)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	508.84	501.73	494.94	488.36	481.93	475.65	469.49	463.43	457.48
0	5	491.68	485.33	479.24	473.32	467.53	461.85	456.27	450.79	445.38
0	10	478.08	472.31	466.77	461.35	456.05	450.84	445.70	440.65	435.66
0	15	467.32	462.02	456.89	451.88	446.95	442.10	437.31	432.58	427.91
0	20	458.91	453.96	449.17	444.46	439.83	435.26	430.75	426.29	421.88
0	25	452.46	447.79	443.24	438.78	434.39	430.05	425.77	421.53	417.33
0	30	447.69	443.22	438.87	434.60	430.39	426.24	422.13	418.08	414.06
0	35	444.38	440.03	435.81	431.67	427.60	423.59	419.63	415.71	411.84
0	40	442.37	438.06	433.89	429.81	425.81	421.87	418.00	414.18	410.40
10	0	494.63	487.94	481.49	475.21	469.07	463.06	457.18	451.41	445.76
10	5	478.52	472.52	466.70	461.02	455.46	450.01	444.65	439.40	434.25
10	10	465.65	460.20	454.88	449.67	444.55	439.53	434.60	429.74	424.97
10	15	455.41	450.38	445.46	440.63	435.87	431.19	426.58	422.04	417.57
10	20	447.33	442.65	438.05	433.51	429.04	424.64	420.30	416.02	411.80
10	25	441.09	436.68	432.33	428.04	423.80	419.63	415.51	411.44	407.44
10	30	436.43	432.22	428.07	423.97	419.92	415.93	411.99	408.11	404.29
10	35	433.15	429.08	425.06	421.10	417.19	413.34	409.55	405.81	402.13
10	40	431.09	427.08	423.13	419.24	415.41	411.64	407.94	404.29	400.71
20	0	480.97	474.67	468.52	462.51	456.63	450.88	445.26	439.75	434.37
20	5	465.86	460.19	454.62	449.16	443.81	438.56	433.42	428.39	423.47
20	10	453.70	448.53	443.43	438.41	433.47	428.63	423.88	419.22	414.66
20	15	443.95	439.19	434.46	429.80	425.20	420.68	416.24	411.89	407.62
20	20	436.22	431.79	427.37	422.99	418.67	414.42	410.24	406.13	402.10
20	25	430.19	426.03	421.86	417.72	413.63	409.60	405.63	401.74	397.91
20	30	425.66	421.70	417.72	413.77	409.87	406.02	402.23	398.51	394.86
20	35	422.43	418.61	414.77	410.96	407.19	403.48	399.83	396.25	392.73
20	40	420.35	416.60	412.83	409.10	405.41	401.78	398.22	394.72	391.31
40	0	455.14	449.55	443.95	438.42	433.00	427.69	422.52	417.50	412.62
40	5	441.91	436.84	431.72	426.65	421.67	416.80	412.05	407.43	402.94
40	10	431.11	426.46	421.75	417.07	412.46	407.94	403.53	399.24	395.08
40	15	422.33	418.05	413.67	409.31	405.01	400.79	396.66	392.65	388.76
40	20	415.28	411.30	407.21	403.11	399.07	395.09	391.21	387.42	383.76
40	25	409.73	406.00	402.13	398.26	394.42	390.65	386.96	383.37	379.89
40	30	405.48	401.95	398.26	394.56	390.89	387.28	383.75	380.32	376.99
40	35	402.39	398.99	395.44	391.87	388.32	384.83	381.41	378.09	374.87
40	40	400.33	397.01	393.53	390.02	386.55	383.12	379.77	376.51	373.36
100	0	388.25	384.41	380.13	375.78	371.47	367.29	363.30	359.52	356.00
100	5	379.67	376.09	372.08	367.99	363.96	360.05	356.34	352.85	349.61
100	10	372.39	369.06	365.30	361.46	357.68	354.04	350.57	347.34	344.36
100	15	366.29	363.19	359.65	356.03	352.47	349.04	345.79	342.77	340.00
100	20	361.24	358.34	354.99	351.55	348.16	344.90	341.82	338.96	336.34
100	25	357.15	354.42	351.21	347.91	344.65	341.51	338.54	335.78	333.27
100	30	353.92	351.33	348.23	345.03	341.85	338.78	335.87	333.16	330.69
100	35	351.49	349.00	345.99	342.85	339.71	336.68	333.79	331.09	328.61
100	40	349.77	347.38	344.44	341.34	338.23	335.19	332.30	329.57	327.06

Table 24
Adiabatic compressibility, $\kappa(S, t, p)$, in ppm/MPa, Eq. (9)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	508.54	501.58	494.89	488.35	481.93	475.62	469.39	463.24	457.18
0	5	491.66	485.26	479.09	473.06	467.15	461.32	455.58	449.91	444.31
0	10	477.55	471.63	465.91	460.31	454.80	449.38	444.02	438.73	433.50
0	15	465.76	460.23	454.88	449.63	444.47	439.37	434.34	429.36	424.44
0	20	455.91	450.72	445.67	440.72	435.84	431.02	426.26	421.54	416.88
0	25	447.72	442.80	438.01	433.31	428.67	424.08	419.56	415.07	410.64
0	30	440.98	436.29	431.71	427.21	422.77	418.39	414.06	409.77	405.53
0	35	435.51	431.00	426.60	422.27	418.00	413.77	409.59	405.46	401.36
0	40	431.15	426.82	422.56	418.36	414.19	410.07	405.98	401.92	397.89
10	0	494.57	487.93	481.49	475.17	468.97	462.88	456.88	450.98	445.17
10	5	478.39	472.28	466.34	460.51	454.78	449.15	443.61	438.15	432.78
10	10	464.81	459.17	453.65	448.24	442.91	437.66	432.49	427.40	422.38
10	15	453.42	448.17	443.02	437.95	432.95	428.03	423.17	418.38	413.66
10	20	443.88	438.96	434.11	429.33	424.62	419.97	415.38	410.86	406.40
10	25	435.93	431.28	426.69	422.16	417.69	413.28	408.92	404.63	400.40
10	30	429.35	424.93	420.56	416.24	411.98	407.77	403.61	399.52	395.48
10	35	423.99	419.77	415.58	411.44	407.34	403.29	399.28	395.33	391.43
10	40	419.69	415.66	411.62	407.61	403.62	399.67	395.76	391.89	388.05
20	0	480.97	474.64	468.43	462.33	456.33	450.45	444.66	438.99	433.43
20	5	465.50	459.68	453.95	448.31	442.77	437.32	431.97	426.72	421.57
20	10	452.48	447.11	441.79	436.55	431.39	426.31	421.32	416.42	411.61
20	15	441.52	436.53	431.57	426.67	421.83	417.07	412.38	407.78	403.26
20	20	432.31	427.64	422.98	418.37	413.81	409.32	404.90	400.56	396.30
20	25	424.61	420.21	415.81	411.44	407.12	402.87	398.68	394.57	390.53
20	30	418.22	414.05	409.87	405.71	401.60	397.54	393.55	389.63	385.78
20	35	412.98	409.02	405.02	401.04	397.09	393.19	389.34	385.55	381.83
20	40	408.75	404.99	401.15	397.29	393.46	389.65	385.88	382.16	378.49
40	0	454.84	449.10	443.34	437.63	432.02	426.52	421.14	415.90	410.80
40	5	440.87	435.59	430.26	424.98	419.78	414.68	409.69	404.82	400.09
40	10	429.01	424.14	419.20	414.29	409.44	404.68	400.03	395.50	391.09
40	15	418.95	414.44	409.84	405.24	400.71	396.25	391.89	387.65	383.52
40	20	410.44	406.24	401.92	397.61	393.34	389.14	385.04	381.05	377.17
40	25	403.28	399.34	395.27	391.19	387.15	383.18	379.29	375.51	371.84
40	30	397.30	393.59	389.72	385.84	381.98	378.19	374.48	370.87	367.36
40	35	392.36	388.85	385.16	381.43	377.72	374.06	370.47	366.96	363.55
40	40	388.33	385.02	381.48	377.87	374.25	370.65	367.11	363.63	360.23
100	0	385.35	381.22	376.67	372.05	367.49	363.07	358.84	354.84	351.10
100	5	375.68	371.86	367.61	363.30	359.04	354.92	350.99	347.28	343.84
100	10	367.26	363.74	359.77	355.74	351.76	347.92	344.27	340.84	337.67
100	15	359.96	356.70	352.98	349.20	345.46	341.86	338.45	335.26	332.32
100	20	353.67	350.62	347.12	343.53	339.99	336.58	333.35	330.34	327.59
100	25	348.28	345.42	342.08	338.64	335.24	331.96	328.86	325.96	323.32
100	30	343.70	341.01	337.80	334.47	331.16	327.95	324.90	322.06	319.44
100	35	339.85	337.32	334.23	330.98	327.71	324.53	321.47	318.60	315.94
100	40	336.65	334.34	331.37	328.18	324.94	321.73	318.62	315.64	312.85

Table 25
Isothermal haline contraction coefficient, $\beta(S, t, p)$, in ppm, Eq. (8)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	827.49	806.75	800.58	796.35	792.97	790.01	787.28	784.65	782.06
0	5	807.85	789.37	784.16	780.71	777.99	775.63	773.45	771.35	769.26
0	10	792.85	775.68	771.07	768.09	765.80	763.83	762.02	760.27	758.51
0	15	781.15	764.79	760.58	757.94	755.94	754.25	752.71	751.21	749.71
0	20	771.85	756.06	752.14	749.76	748.00	746.54	745.22	743.94	742.66
0	25	764.48	749.06	745.37	743.20	741.64	740.38	739.25	738.16	737.06
0	30	758.97	743.64	740.08	738.03	736.60	735.46	734.47	733.52	732.56
0	35	755.71	739.90	736.22	734.11	732.66	731.53	730.54	729.62	728.70
0	40	755.48	738.16	733.94	731.45	729.70	728.32	727.13	726.03	724.94
10	0	813.03	793.30	787.54	783.65	780.55	777.85	775.35	772.94	770.55
10	5	794.98	777.28	772.42	769.24	766.75	764.60	762.62	760.70	758.79
10	10	781.23	764.67	760.34	757.58	755.48	753.68	752.03	750.43	748.83
10	15	770.52	754.63	750.64	748.17	746.33	744.79	743.38	742.01	740.64
10	20	762.00	746.56	742.83	740.59	738.97	737.63	736.43	735.26	734.08
10	25	755.22	740.09	736.56	734.52	733.08	731.92	730.90	729.91	728.91
10	30	750.13	735.07	731.65	729.72	728.40	727.38	726.48	725.63	724.77
10	35	747.12	731.59	728.05	726.08	724.74	723.72	722.84	722.02	721.20
10	40	746.95	729.96	725.90	723.56	721.95	720.69	719.61	718.62	717.65
20	0	799.67	780.53	775.11	771.51	768.67	766.22	763.96	761.79	759.64
20	5	783.05	765.77	761.17	758.21	755.95	754.01	752.24	750.53	748.81
20	10	770.46	754.16	750.01	747.44	745.52	743.91	742.43	741.01	739.58
20	15	760.69	744.92	741.06	738.75	737.06	735.66	734.41	733.20	731.98
20	20	752.91	737.49	733.86	731.74	730.25	729.04	727.97	726.94	725.91
20	25	746.71	731.54	728.08	726.14	724.81	723.78	722.87	722.02	721.15
20	30	742.06	726.91	723.56	721.72	720.51	719.59	718.82	718.09	717.36
20	35	739.32	723.70	720.24	718.35	717.13	716.22	715.46	714.77	714.08
20	40	739.26	722.20	718.23	716.00	714.49	713.36	712.41	711.56	710.72
40	0	776.00	756.88	751.85	748.69	746.34	744.39	742.65	741.02	739.41
40	5	761.87	744.30	739.96	737.37	735.51	734.01	732.70	731.48	730.27
40	10	751.34	734.47	730.49	728.20	726.62	725.39	724.33	723.35	722.38
40	15	743.27	726.71	722.93	720.82	719.42	718.36	717.48	716.67	715.88
40	20	736.88	720.49	716.86	714.91	713.66	712.75	712.02	711.36	710.72
40	25	731.80	715.52	712.02	710.20	709.09	708.32	707.74	707.23	706.74
40	30	728.00	711.67	708.23	706.50	705.48	704.83	704.35	703.96	703.60
40	35	725.85	709.01	705.45	703.66	702.62	701.97	701.51	701.15	700.81
40	40	726.08	707.81	703.75	701.61	700.30	699.42	698.77	698.24	697.77
100	0	726.57	698.47	692.58	689.92	688.77	688.48	688.74	689.37	690.24
100	5	717.28	690.38	685.08	682.89	682.16	682.27	682.91	683.90	685.12
100	10	711.11	684.55	679.46	677.48	676.96	677.26	678.09	679.26	680.67
100	15	706.88	680.25	675.22	673.31	672.88	673.28	674.21	675.50	677.02
100	20	703.80	676.95	671.92	670.06	669.67	670.14	671.15	672.52	674.14
100	25	701.47	674.36	669.30	667.45	667.11	667.64	668.71	670.16	671.86
100	30	699.91	672.35	667.20	665.31	664.97	665.51	666.61	668.10	669.85
100	35	699.49	671.02	665.58	663.51	663.03	663.47	664.50	665.94	667.65
100	40	701.00	670.66	664.55	661.99	661.12	661.23	661.98	663.16	664.64

Table 26
Adiabatic haline contraction coefficient, β' (S, t, p), in ppm, Eq. (11)

MPa	°C	S = 0	S = 5	S = 10	S = 15	S = 20	S = 25	S = 30	S = 35	S = 40
0	0	827.49	805.91	800.41	796.37	792.92	789.73	786.65	783.57	780.45
0	5	807.85	789.68	784.06	780.12	776.84	773.86	770.99	768.15	765.28
0	10	792.85	775.90	769.98	765.97	762.71	759.81	757.05	754.34	751.61
0	15	781.15	763.97	757.67	753.53	750.25	747.37	744.67	742.04	739.42
0	20	771.85	753.42	746.75	742.47	739.15	736.28	733.63	731.08	728.55
0	25	764.48	743.99	736.94	732.53	729.15	726.29	723.68	721.19	718.74
0	30	758.97	735.60	728.12	723.53	720.07	717.16	714.55	712.07	709.67
0	35	755.71	728.41	720.34	715.43	711.76	708.71	705.97	703.40	700.91
0	40	755.48	722.82	713.82	708.31	704.20	700.77	697.70	694.81	692.02
10	0	813.03	793.11	787.57	783.57	780.19	777.09	774.11	771.14	768.13
10	5	794.98	777.77	772.13	768.24	765.04	762.15	759.39	756.65	753.89
10	10	781.23	764.76	758.83	754.87	751.70	748.88	746.22	743.60	740.98
10	15	770.52	753.49	747.20	743.11	739.90	737.10	734.49	731.96	729.44
10	20	762.00	743.53	736.86	732.64	729.39	726.60	724.04	721.57	719.14
10	25	755.22	734.59	727.56	723.21	719.92	717.13	714.61	712.21	709.86
10	30	750.13	726.62	719.19	714.66	711.29	708.47	705.95	703.57	701.26
10	35	747.12	719.79	711.78	706.95	703.37	700.40	697.76	695.28	692.89
10	40	746.95	714.47	705.54	700.11	696.08	692.73	689.75	686.95	684.24
20	0	799.67	780.82	775.17	771.20	767.90	764.90	762.04	759.21	756.36
20	5	783.05	766.32	760.60	756.74	753.62	750.83	748.19	745.60	742.98
20	10	770.46	754.05	748.05	744.13	741.03	738.31	735.77	733.28	730.80
20	15	760.69	743.45	737.08	733.03	729.89	727.18	724.69	722.28	719.89
20	20	752.91	734.05	727.32	723.14	719.96	717.26	714.80	712.46	710.16
20	25	746.71	725.61	718.54	714.22	711.00	708.31	705.90	703.62	701.40
20	30	742.06	718.08	710.61	706.13	702.83	700.11	697.69	695.44	693.25
20	35	739.32	711.60	703.58	698.79	695.29	692.42	689.88	687.53	685.25
20	40	739.26	706.58	697.62	692.24	688.28	685.02	682.13	679.43	676.83
40	0	776.00	757.67	751.64	747.66	744.51	741.78	739.24	736.80	734.37
40	5	761.87	744.74	738.68	734.82	731.86	729.33	727.01	724.78	722.58
40	10	751.34	733.89	727.56	723.64	720.70	718.22	715.98	713.86	711.77
40	15	743.27	724.54	717.86	713.81	710.81	708.35	706.15	704.09	702.08
40	20	736.88	716.26	709.22	705.04	702.00	699.54	697.38	695.38	693.46
40	25	731.80	708.79	701.43	697.12	694.04	691.58	689.46	687.52	685.68
40	30	728.00	702.11	694.37	689.89	686.74	684.24	682.11	680.19	678.37
40	35	725.85	696.36	688.06	683.27	679.90	677.24	674.98	672.93	671.01
40	40	726.08	691.91	682.67	677.25	673.39	670.30	667.64	665.22	662.92
100	0	726.57	698.68	690.19	685.80	683.25	681.73	680.85	680.41	680.27
100	5	717.28	689.55	681.15	676.94	674.60	673.31	672.66	672.45	672.53
100	10	711.11	682.28	673.68	669.44	667.14	665.91	665.35	665.23	665.42
100	15	706.88	676.16	667.27	662.91	660.56	659.33	658.80	658.73	658.99
100	20	703.80	670.71	661.50	657.01	654.60	653.35	652.83	652.79	653.10
100	25	701.47	665.68	656.14	651.49	648.99	647.70	647.17	647.14	647.46
100	30	699.91	661.07	651.09	646.19	643.53	642.12	641.48	641.37	641.62
100	35	699.49	657.09	646.41	641.05	638.02	636.30	635.38	634.99	634.98
100	40	701.00	654.22	642.30	636.07	632.32	629.94	628.40	627.41	626.78

Table 27
Adiabatic lapse rate, $\Gamma(S, t, p)$, in mK/MPa, Eq. (10)

MPa	°C	$S = 0$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 25$	$S = 30$	$S = 35$	$S = 40$
0	0	−4.386	−3.069	−1.845	−0.681	0.434	1.506	2.537	3.532	4.491
0	5	1.061	2.124	3.132	4.102	5.041	5.953	6.839	7.701	8.541
0	10	5.937	6.808	7.642	8.452	9.240	10.010	10.763	11.498	12.216
0	15	10.387	11.111	11.808	12.487	13.150	13.797	14.430	15.048	15.652
0	20	14.515	15.124	15.712	16.283	16.840	17.383	17.913	18.429	18.933
0	25	18.400	18.907	19.399	19.879	20.347	20.803	21.248	21.682	22.104
0	30	22.099	22.497	22.894	23.289	23.678	24.063	24.442	24.815	25.182
0	35	25.657	25.915	26.203	26.508	26.825	27.150	27.482	27.820	28.162
0	40	29.109	29.173	29.321	29.521	29.761	30.035	30.339	30.670	31.025
10	0	−1.989	−0.758	0.384	1.469	2.506	3.502	4.459	5.381	6.268
10	5	3.034	4.025	4.964	5.868	6.744	7.595	8.422	9.226	10.008
10	10	7.557	8.366	9.144	9.900	10.637	11.357	12.060	12.748	13.421
10	15	11.706	12.379	13.030	13.664	14.284	14.889	15.481	16.060	16.626
10	20	15.574	16.142	16.690	17.224	17.745	18.252	18.747	19.230	19.700
10	25	19.229	19.702	20.162	20.611	21.048	21.474	21.889	22.293	22.686
10	30	22.721	23.092	23.464	23.833	24.198	24.558	24.912	25.261	25.603
10	35	26.090	26.329	26.599	26.885	27.184	27.491	27.806	28.126	28.451
10	40	29.369	29.421	29.560	29.750	29.982	30.249	30.546	30.869	31.218
20	0	0.278	1.428	2.494	3.505	4.471	5.396	6.285	7.139	7.960
20	5	4.910	5.831	6.705	7.548	8.365	9.158	9.928	10.678	11.407
20	10	9.103	9.854	10.577	11.281	11.968	12.640	13.297	13.940	14.569
20	15	12.971	13.595	14.200	14.791	15.368	15.933	16.485	17.026	17.554
20	20	16.594	17.120	17.631	18.128	18.612	19.085	19.546	19.995	20.432
20	25	20.031	20.471	20.899	21.316	21.722	22.119	22.504	22.879	23.244
20	30	23.328	23.672	24.017	24.361	24.700	25.036	25.366	25.690	26.009
20	35	26.518	26.736	26.987	27.254	27.535	27.824	28.122	28.425	28.733
20	40	29.632	29.671	29.798	29.979	30.202	30.461	30.750	31.068	31.411
40	0	4.423	5.428	6.356	7.234	8.071	8.871	9.637	10.371	11.076
40	5	8.364	9.158	9.914	10.643	11.350	12.036	12.703	13.352	13.983
40	10	11.971	12.611	13.233	13.840	14.433	15.014	15.583	16.140	16.685
40	15	15.331	15.862	16.380	16.887	17.384	17.871	18.347	18.813	19.269
40	20	18.509	18.956	19.392	19.818	20.233	20.638	21.033	21.418	21.792
40	25	21.548	21.922	22.287	22.644	22.992	23.331	23.660	23.981	24.291
40	30	24.485	24.774	25.069	25.364	25.656	25.946	26.231	26.512	26.788
40	35	27.348	27.523	27.734	27.966	28.213	28.471	28.738	29.013	29.295
40	40	30.158	30.166	30.270	30.431	30.638	30.884	31.164	31.474	31.813
100	0	13.938	14.618	15.237	15.814	16.356	16.868	17.350	17.806	18.236
100	5	16.452	16.947	17.419	17.873	18.310	18.732	19.139	19.532	19.910
100	10	18.797	19.171	19.540	19.901	20.255	20.602	20.939	21.269	21.589
100	15	21.030	21.329	21.628	21.925	22.216	22.501	22.780	23.052	23.316
100	20	23.190	23.441	23.694	23.944	24.190	24.431	24.666	24.895	25.116
100	25	25.305	25.514	25.730	25.948	26.165	26.380	26.591	26.798	27.000
100	30	27.396	27.547	27.725	27.917	28.118	28.325	28.536	28.751	28.968
100	35	29.481	29.536	29.659	29.822	30.015	30.235	30.476	30.738	31.017
100	40	31.567	31.463	31.499	31.623	31.816	32.068	32.373	32.728	33.129

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Corrigendum

Corrigendum to: A new extended Gibbs thermodynamic potential of seawater [Progress in Oceanography 58 (2003) 43–114] ☆

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Page 46: *The sentence*

The recently published equation of state of McDougall et al. (2003) is a computationally faster but numerically equivalent formulation of FH95 for use in numerical models, restricted to naturally occurring combinations of salinity, temperature and pressure, so-called 'Neptunian' waters, and formulated in terms of potential density as function of salinity, potential temperature, and pressure.

must read

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Page 82: *Eq.* (53)

$$S = 1.004867 \cdot c/(\mathrm{g/kg})$$

must read

$$c/(\mathrm{g/kg}) = 1.004867 \cdot S$$

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